Principle of EE1

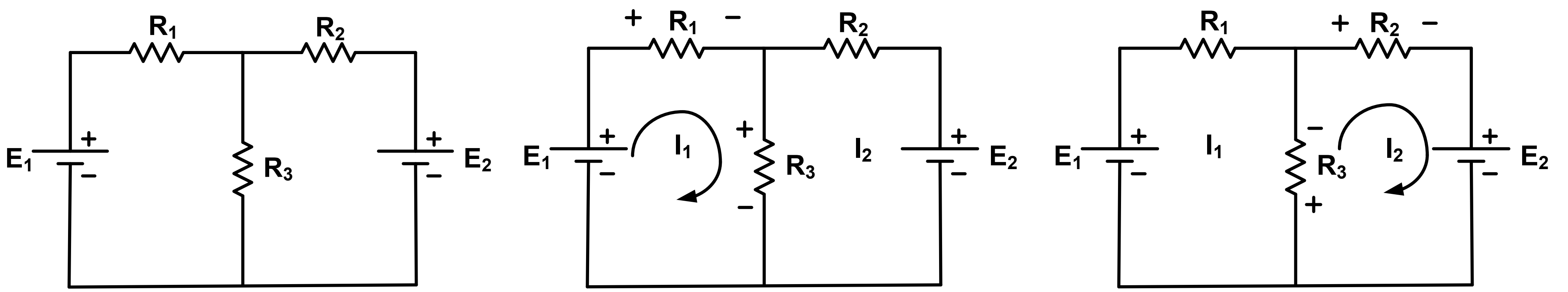
Lesson 3

Prof. Võ Văn Tới
School of Biomedical Engineering
Vice-Provost for Life and Health Science, Engineering and Technology Development
International University
Vietnam National Universities – HCMC

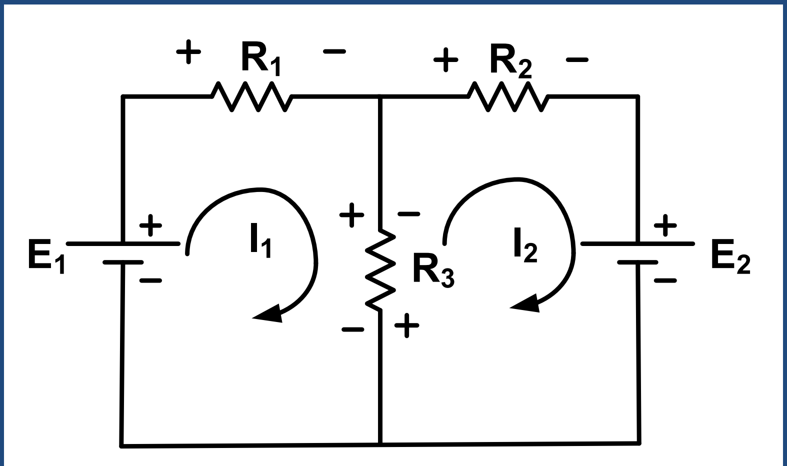
OTHER METHODS OF ANALYZING RESISTIVE CIRCUITS

1. MESH CURRENT ANALYSIS

Principle



Principle



Kirchhoff's voltage laws for each mesh:

Mesh 1: $-E_1 + R_1 I_1 + R_3 I_1 - R_3 I_2 = 0$ (1)

Mesh 2: $R_3 I_2 + R_2 I_2 + E_2 - R_3 I_1 = 0$ (2)

(1) => $(R_1 + R_3)\, I_1 - R_3\, I_2 = E_1$ (1')

(2) => $-R_3\, I_1 + (R_3 + R_2)\, I_2 = -E_2$ (2')

Statement: ΣV of the loads = ΣV of the sources ⇔ Ohm's law

(1'): Sum of resistances in mesh 1 * current of mesh 1 – Common resistance * current of the mesh 2 = Voltage of the source of mesh 1

(2'): Sum of resistances in mesh 2 * current of mesh 2 – Common resistance * current of the mesh 1 = – Voltage of the source of mesh 2

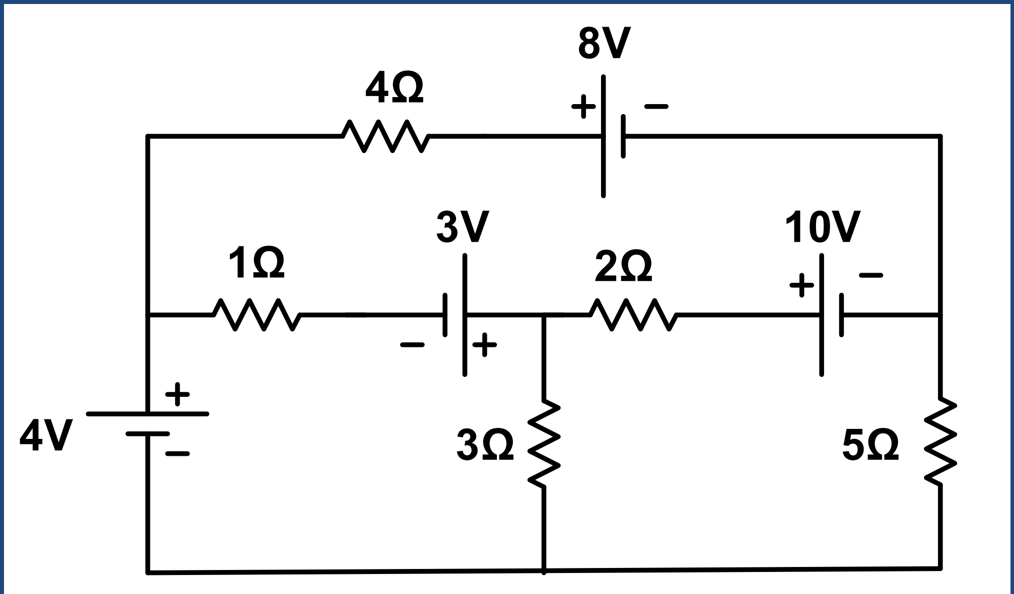
Rules:

1. Select the currents for each mesh: They must be in the same direction
2. Establish equation like (1') and (2') for each mesh. Attention: relation between mesh current direction and source voltage polarity

Situations and corresponding methods

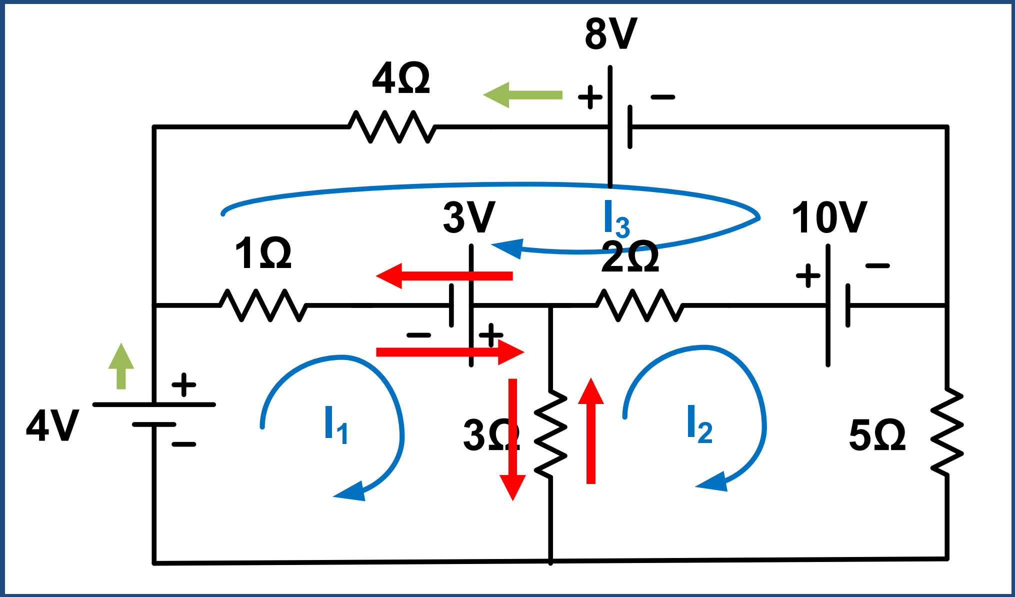
i. Circuits with only voltage sources
ii. Circuits with voltage source and current sources in the outside meshes
iii. Circuits with voltage source and current source between the meshes => Super-mesh

Example 1



Determine the currents through 4V, 8V and 3V sources, and 3Ω resistor

Solution of example 1



1. Give currents for each mesh
2. Equation of each mesh

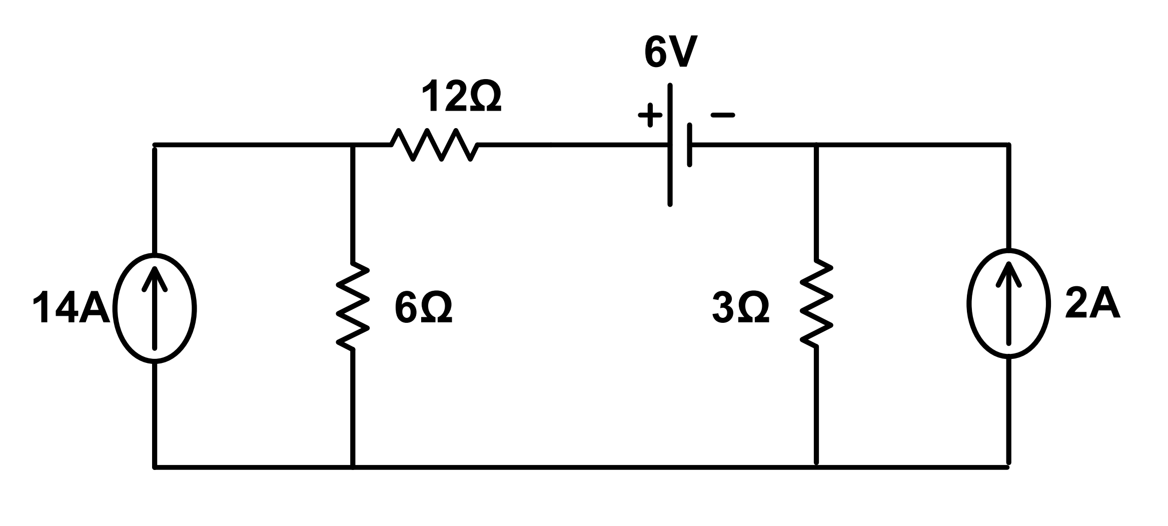
   M1: $(1+3)I_1 - 3I_2 - I_3 = 4 + 3$ (1)

   M2: $-3I_1 + (3+2+5)I_2 - 2I_3 = -10$ (2)

   M3: $-I_1 - 2I_2 + (4+1+2)I_3 = 10 - 3 - 8$ (3)

3. => $I_1 = 1.2A$; $I_2 = -0.67A$; $I_3 = -0.16A$
4. Currents through 4V = $I_1 = 1.2A$ ↑
5. Currents through 8V = $I_3 = -0.16A$ or Currents through 8V is 0.16A ←
6. Currents through 3V $= I_1 - I_3 = 1.2 + 0.16 = 1.36A$ →
7. Currents through 3Ω $= I_1 - I_2 = 1.2 + 0.67 = 1.87$ ↓
   A

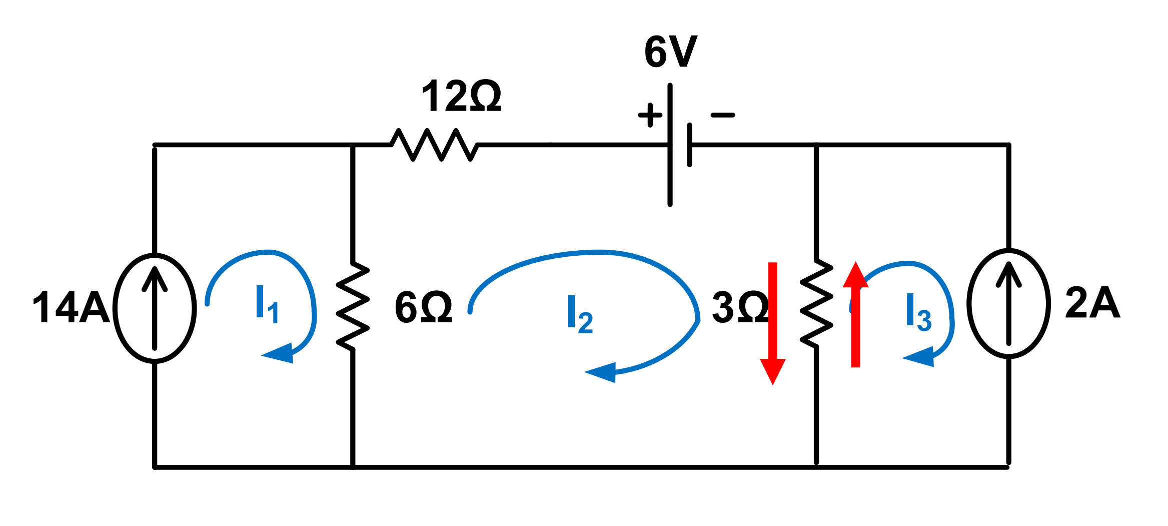
Example 2



Determine the currents through 3Ω resistor

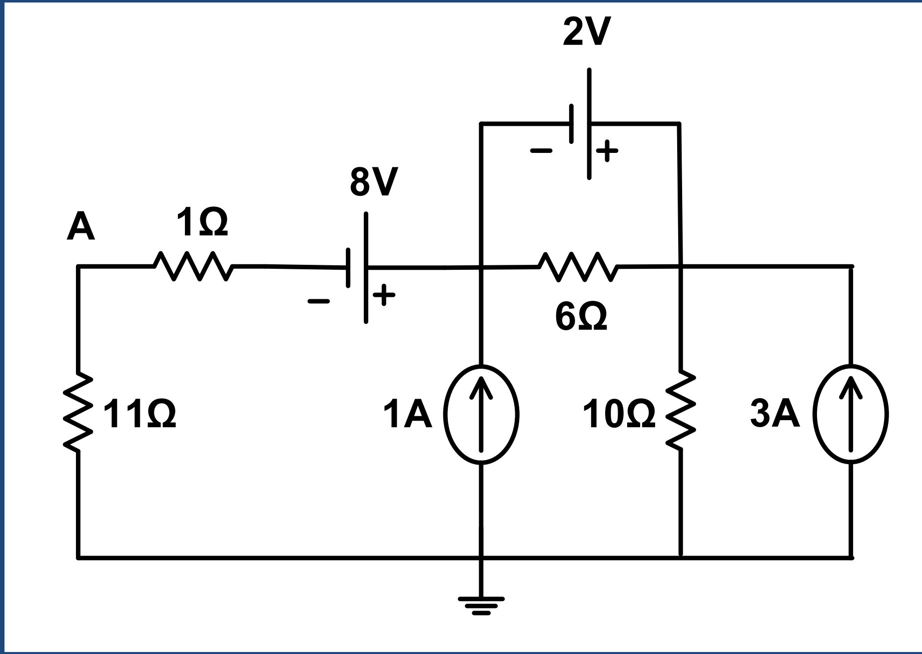
Note: there are current sources in outside meshes

Solution of example 2



1. Give currents for each mesh
2. Equation of each mesh
   - M1: $I_1 = 14$ (1)
   - M2: $-6I_1 + (6 + 12 + 3)I_2 - 3I_3 = -6$ (2)
   - M3: $I_3 = -2$ (3)
3. $\Rightarrow I_2 = 4$ A
4. Currents through 3Ω $= I_2 - I_3 = 4 - (-2) = 6A$ ↓

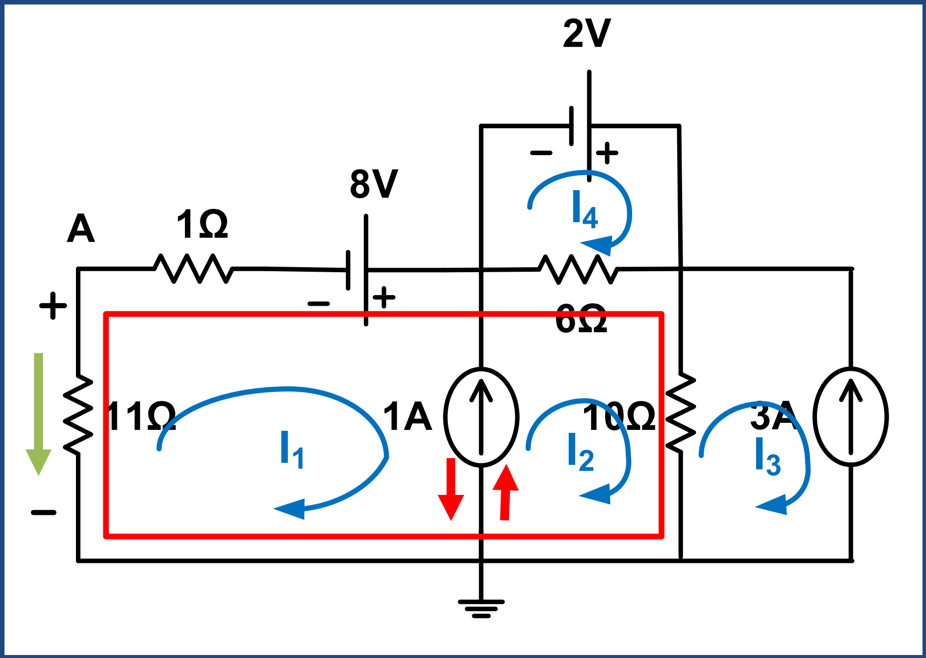
Example 3



Determine the voltage V_A

Note: there is a current source between 2 meshes => super-mesh

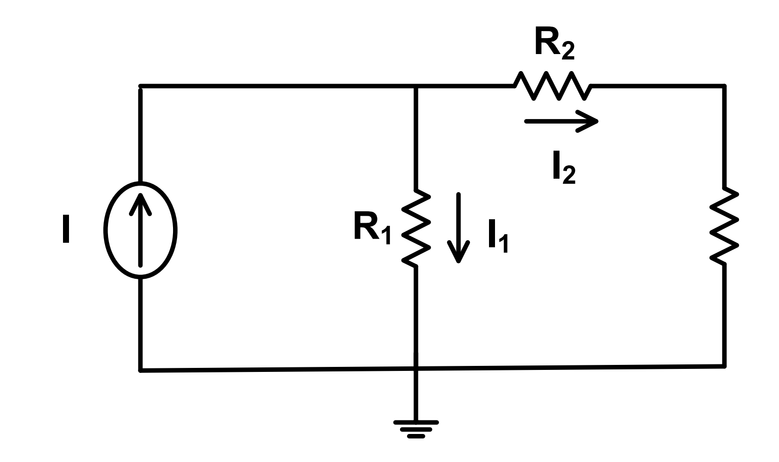
Solution of example 3



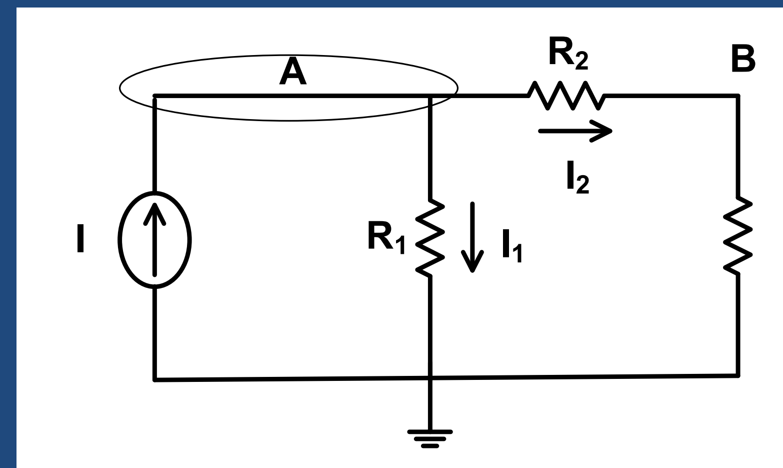
1. Give currents for each mesh
2. Supermesh (1) and (2)
3. Equation of each mesh
   - M1 + M2: $(11+1)I_1 + (6+10)I_2 - 10I_3 - 6I_4 = 8$ (1)
   - M3: $I_3 = -3$ (2)
   - M4: $-6I_2 + 6I_4 = 2$ (3)
   - $I_2 - I_1 = 1$ (4)
4. $\Rightarrow I_1 = -1.4A;\ I_2 = -0.4A;\ I_4 = -0.07A$
5. $V_A = 11 \times 1.4 =$ **15V**

2. NODE VOLTAGE METHOD

Principle



Principle



Kirchhoff's curent laws for node A:

$I_1 + I_2 = I$

$\frac{1}{R_1} V_A + \frac{1}{R_2}(V_A - V_B) = I$ (1)

$$\boxed{(1) \Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_A - \frac{1}{R_2} V_B = I \ (1')}$$

Statement: ΣI of the sources = ΣI of the loads ⇔ Ohm's law

Sum of conductance at node A * voltage of A – Common conductance * voltage of B = Current of the source that goes to A

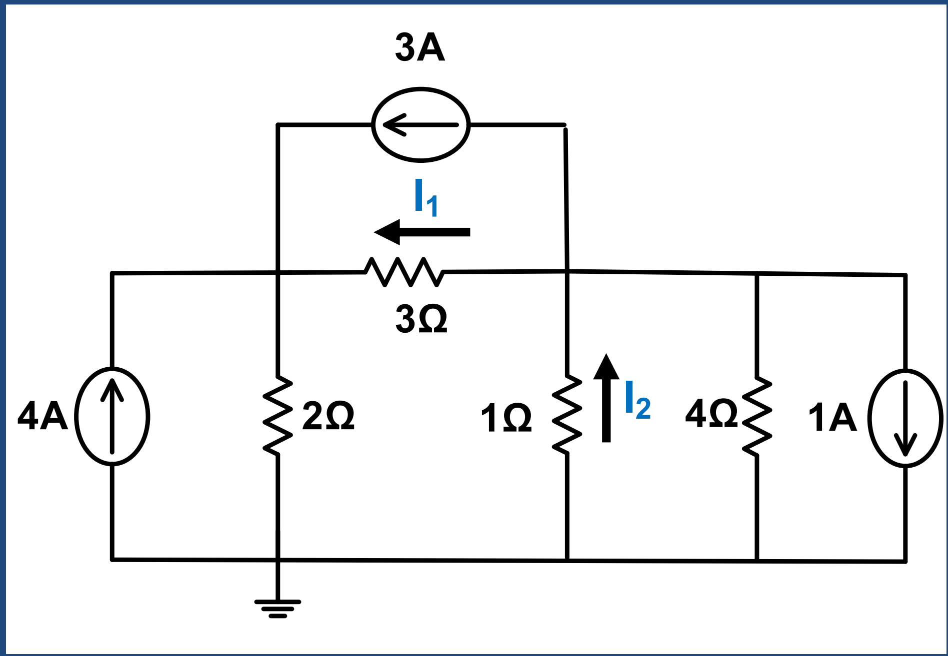
Rules:

1. Identify all nodes and ground the biggest node
2. Establish equation like (1') for each node. Attention: direction of current source: positive if source goes in and negative if source goes out the node

Situations and corresponding methods

i. Circuits with only current sources
ii. Circuits with current sources and voltage source connected to ground
iii. Circuits with current sources and voltage source between 2 nodes => Super-node

Example 4



Determine the currents I_1 and I_2

Solution of example 4

1. Identify all nodes
2. Equation for each node

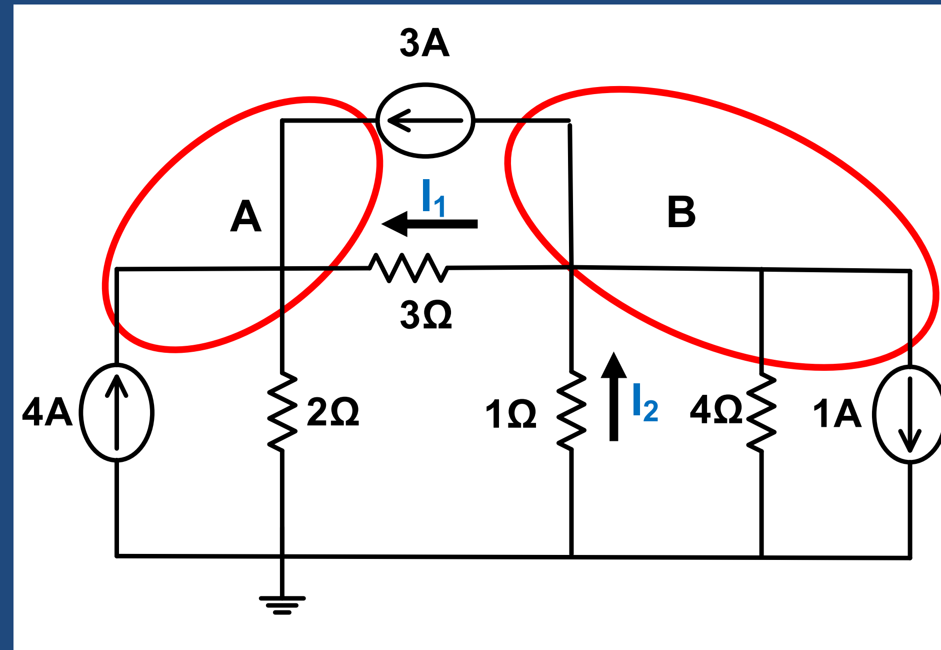
A: $\left(\frac{1}{2}+\frac{1}{3}\right) V_A - \frac{1}{3}V_B = +3$ (1)

B: $-\frac{1}{3}V_A + \left(1+\frac{1}{3}+\frac{1}{4}\right)V_B = -3-1$ (2)

=> $V_A = 8.07V$; $V_B = -0.83V$

=> $I_1 = \frac{V_B - V_A}{3} = \frac{-0.83-8.07}{3} = -\underline{\mathbf{3A}}$

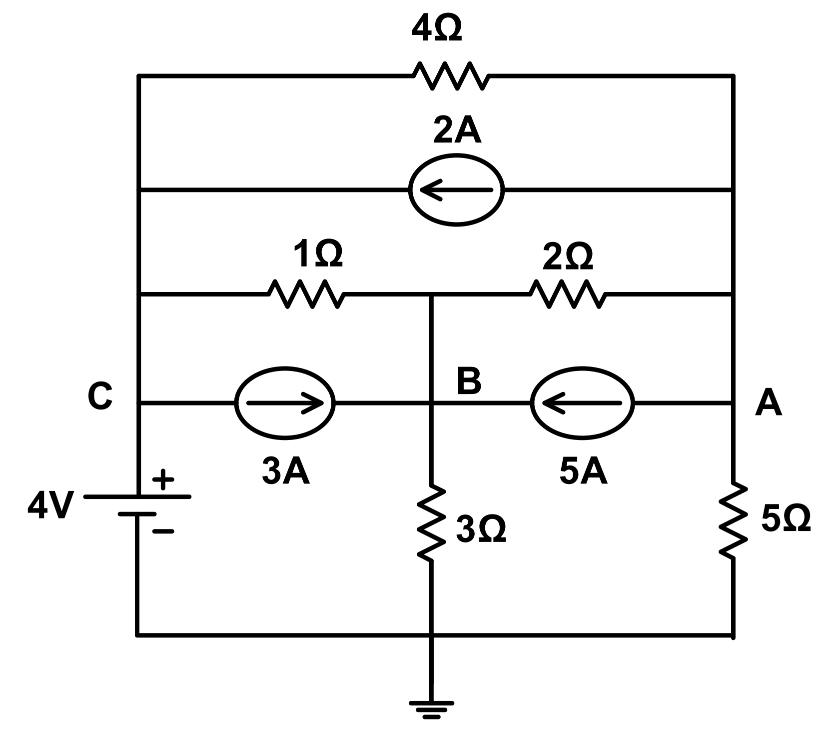
$I_2 = \frac{-V_B}{1} = \underline{\mathbf{0.83A}}$



Example 5

Determine the voltages V_A and V_B

Note: There is a voltage source with one polarity connected to the ground



Solution of example 5

1. Identify all nodes
2. Equation for each node

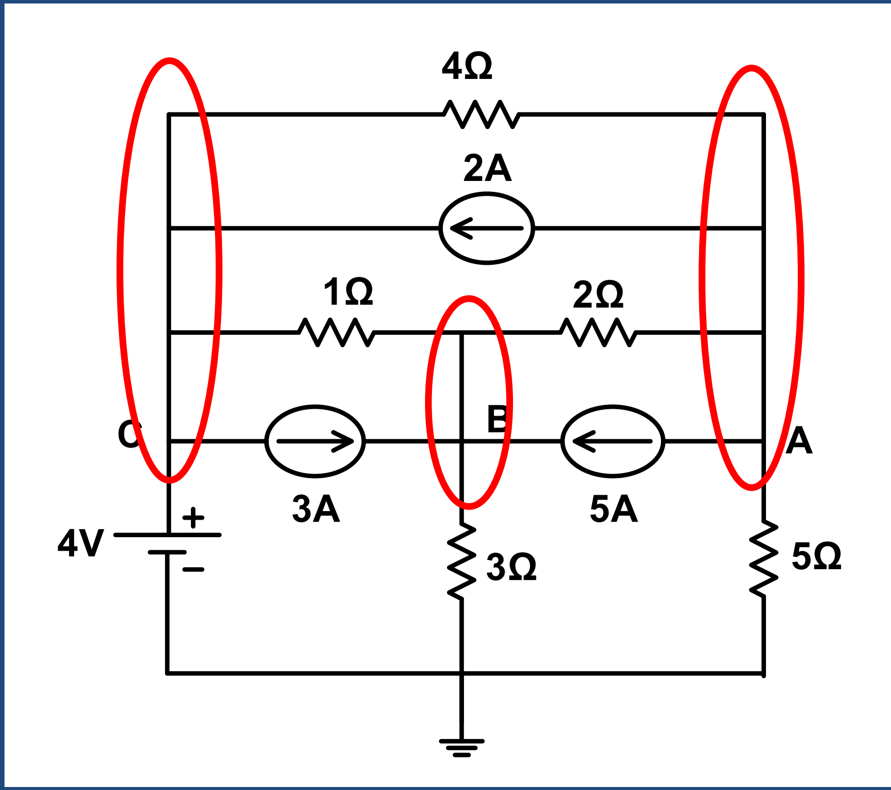
A: $\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{5}\right) V_A - \frac{1}{2}V_B - \frac{1}{4}V_C = -2-5$ (1)

B: $-\frac{1}{2} V_A + \left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}\right)V_B - \frac{1}{1}V_C = 3+5$ (2)

C: $V_C = 4$

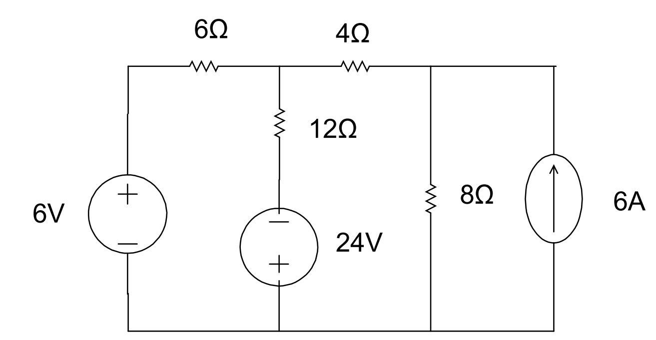
$\Rightarrow V_A = \underline{\mathbf{-3.4V}}$

$V_B = \underline{\mathbf{5.6V}}$



Example 6

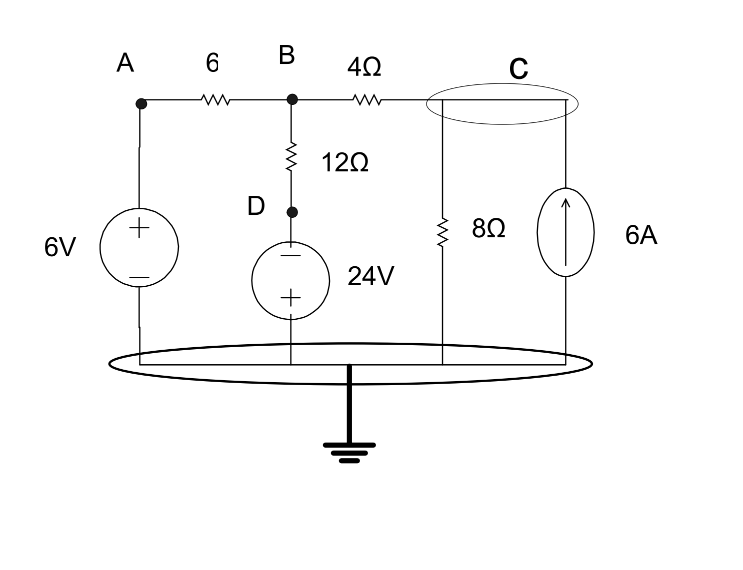
Calculate voltage V across 8Ω and current I through 12Ω



Solution of example 6

Calculate voltage V across 8Ω and current I through 12Ω

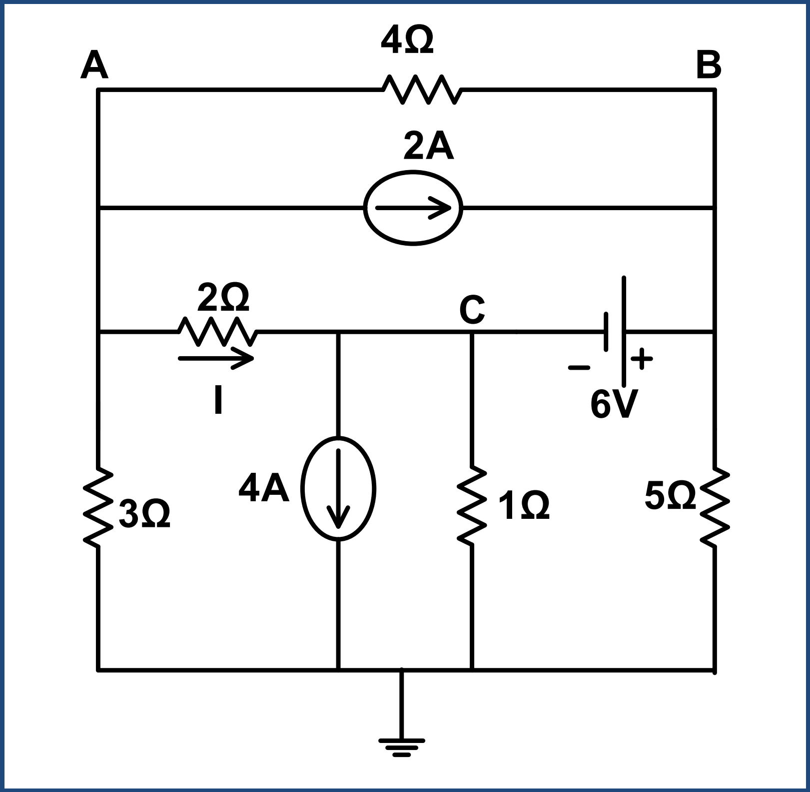
- Identify all nodes
- Ground big node
- $V_A = 6V$
- $V_D = -24V$
- Node B: $-\frac{1}{6}V_A + (\frac{1}{6} + \frac{1}{4} + \frac{1}{12})V_B - \frac{1}{4}V_C - \frac{1}{12}V_D = 0$
- Node C: $-\frac{1}{4}V_B + (\frac{1}{4} + \frac{1}{8})V_C = 6$
- => $V_B = 9V$; $V_C = 22V = V$
- $I = (V_B - V_D)/12 = [9 - (-24)]/12 = 2.75A$



Example 7

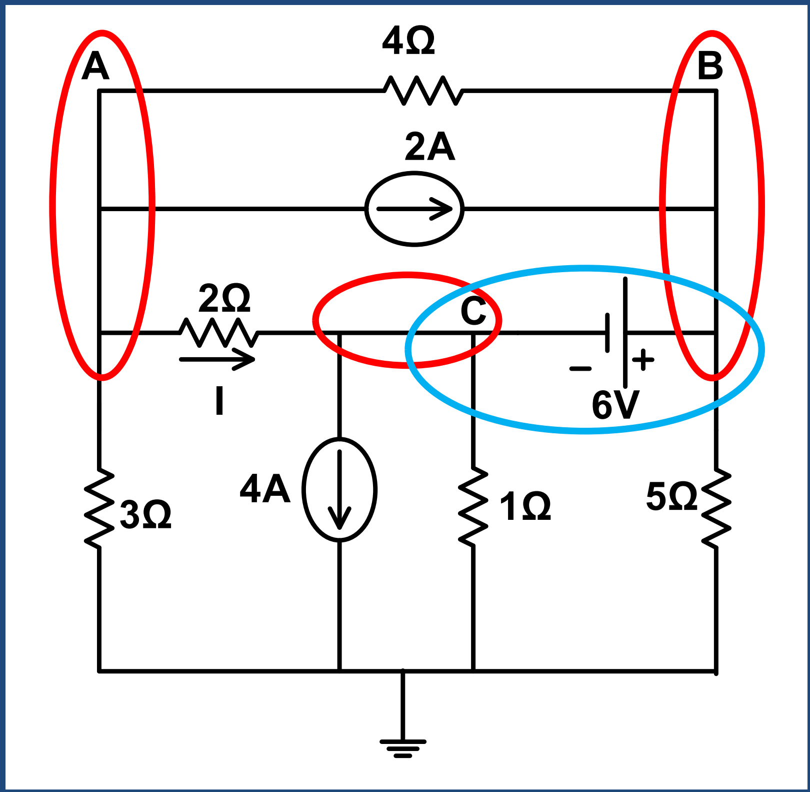
Determine the voltages V_A and current I

Note: There is a voltage source between 2 nodes => super-node



Solution of example 7

1. Identify all nodes
2. Super-node B and C
3. Equation for each node
4. A: $\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{3}\right) V_A - \frac{1}{4}V_B - \frac{1}{2}V_C = -2$ (1)
5. B&C: $-\frac{1}{4}V_A + \left(\frac{1}{4}+\frac{1}{5}\right)V_B + \left(\frac{1}{2}+\frac{1}{1}\right)V_C - \frac{1}{2}V_A = 2 - 4$ (2)
6. C: $V_B - V_C = 6$ (3)



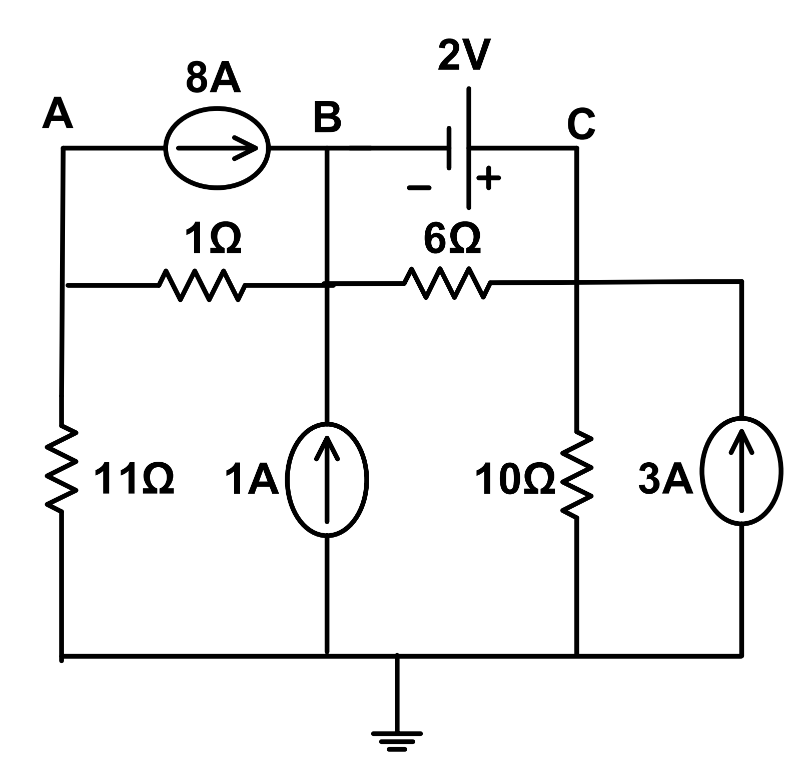
=> $V_A = -2.9V$; $V_B = 2.4V$; $V_C = -3.6V$

$I = (V_A - V_C) / 2 = (-2.9 + 3.6) / 2 =$ **<u>0.35A</u>**

Example 8

Determine the voltage V_A

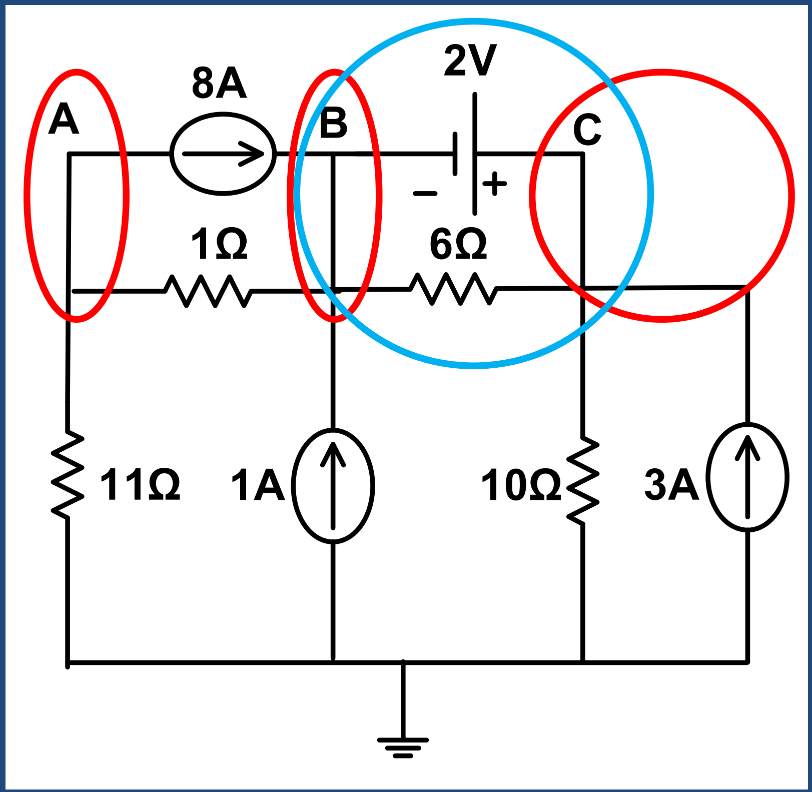
Note: There is a voltage source between 2 nodes => super-node



Solution of example 8

1. Identify all nodes
2. Super-node B and C
3. Equation for each node
4. A: $(\frac{1}{1}+\frac{1}{11})\,V_A - \frac{1}{1}V_B = -8$ (1)
5. B & C: $-\frac{1}{1}V_A + \frac{1}{1}V_B + \frac{1}{10}V_C = 1+8+3$ (2)
6. C: $V_C - V_B = 2$ (3)

$\Rightarrow V_A = \underline{\mathbf{15V}}$



<u>Note</u>: Why did we ignore 6Ω in (2)?

B & C: $-\frac{1}{1}V_A + (\frac{1}{1}+\frac{1}{6})\,V_B - \frac{1}{6}V_C + (\frac{1}{10}+\frac{1}{6})\,V_C - \frac{1}{6}V_B = 1+8+3$

$-\frac{1}{1}V_A + (\frac{1}{1})\,V_B + \frac{1}{10}V_C = 1+8+3$ (2)

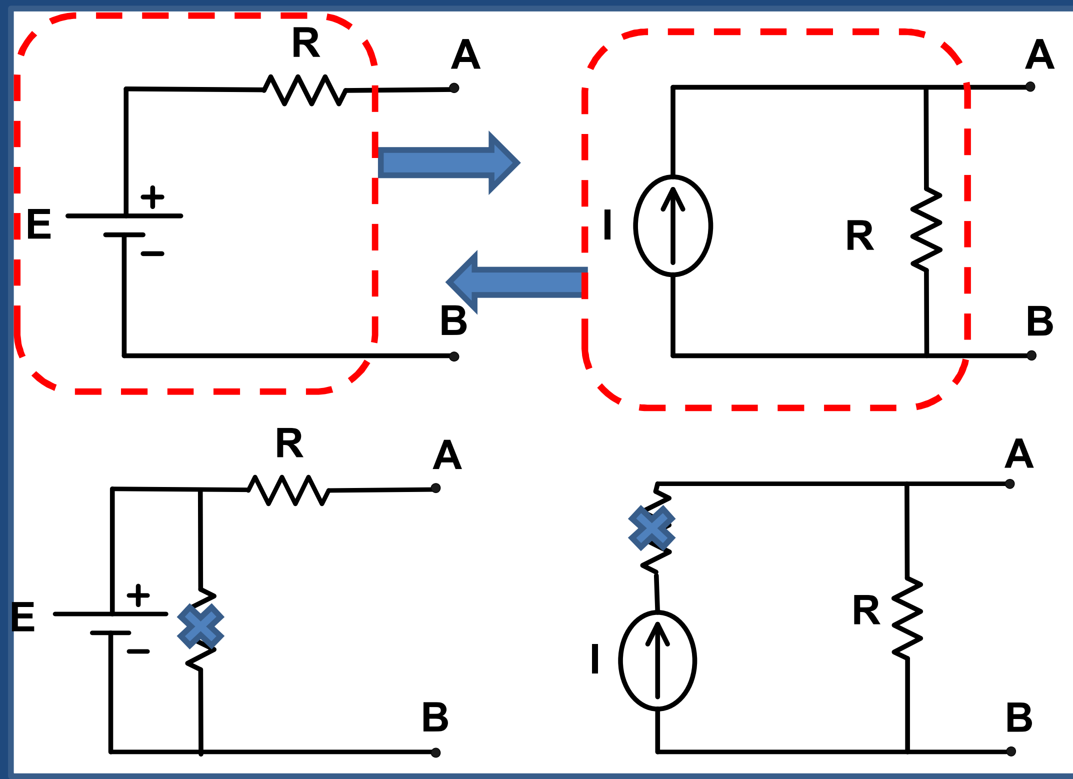
3. SOURCE CONVERSION (TRANSFORMATION) METHOD

Method

a) Thevenin form (with E and R in series) => Norton form: I = E/R and R in parallel

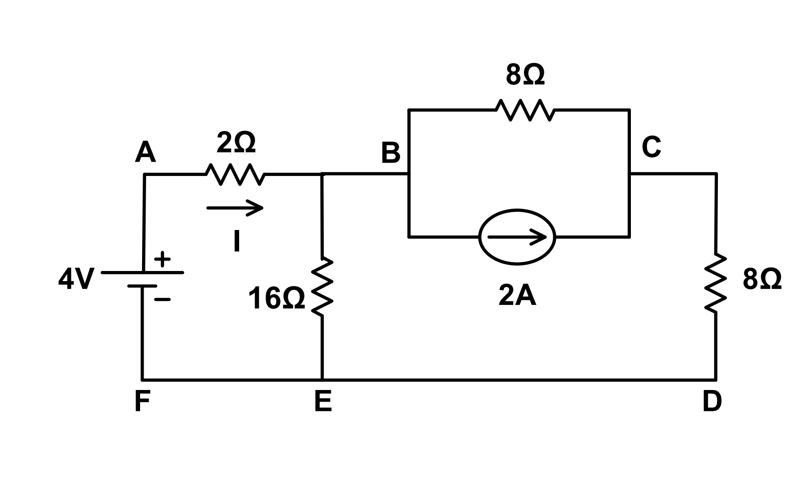
b) Norton form (with I and R in parallel) => Thevenin form: E = IR and R in series

<u>Warning</u>: do not converse the question

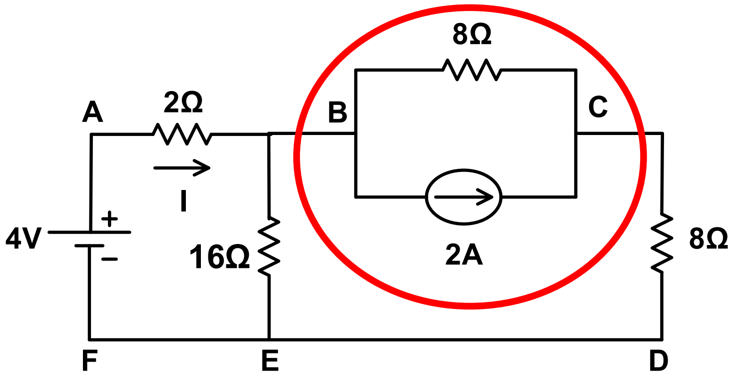


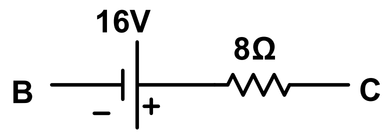
Example 9

Determine the current I

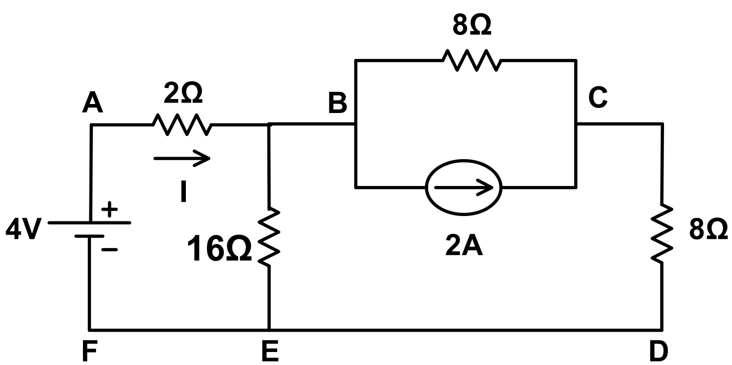


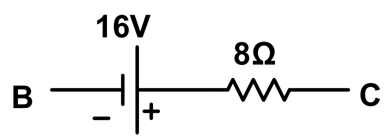
Solution of example 9

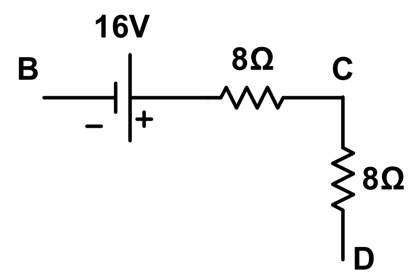


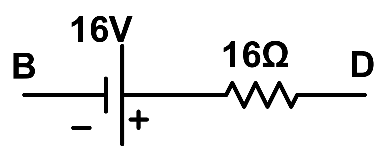


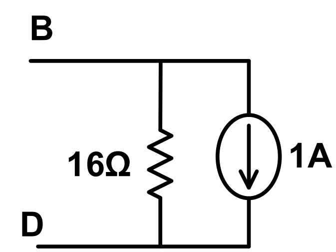
Solution of example 9 (cont.)



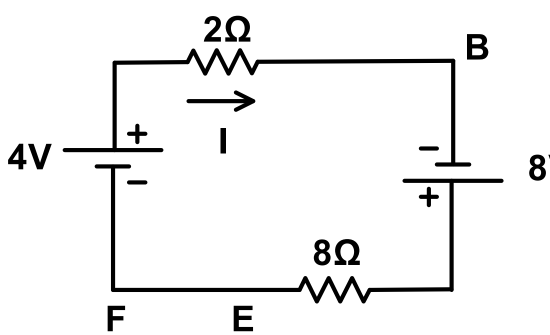


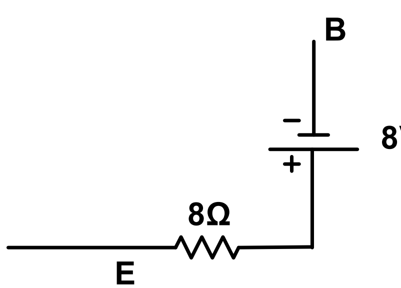


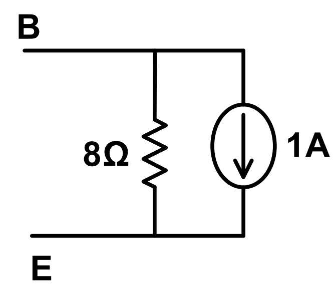


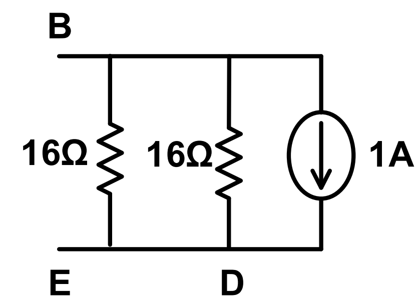


I = (4 + 8) / (2 + 8) = 1.2A



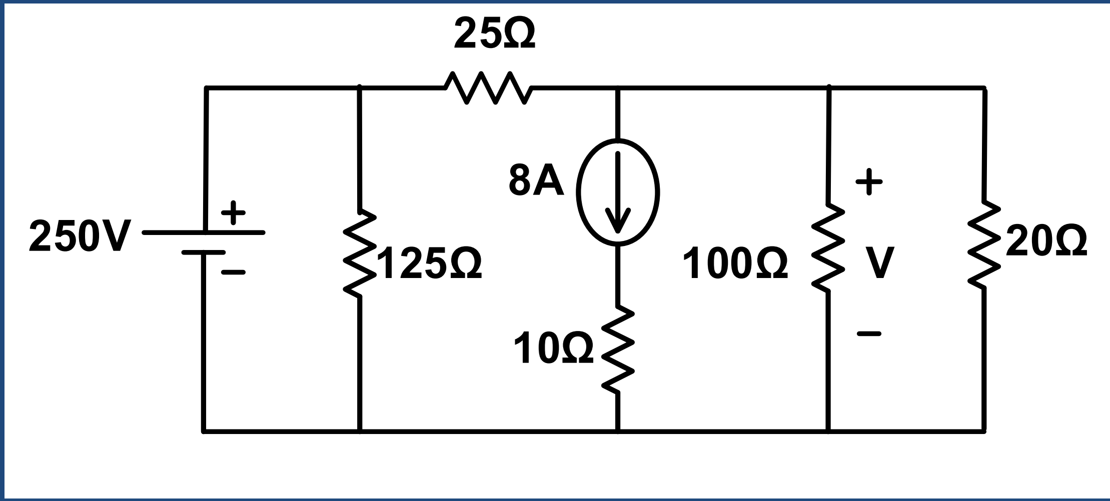




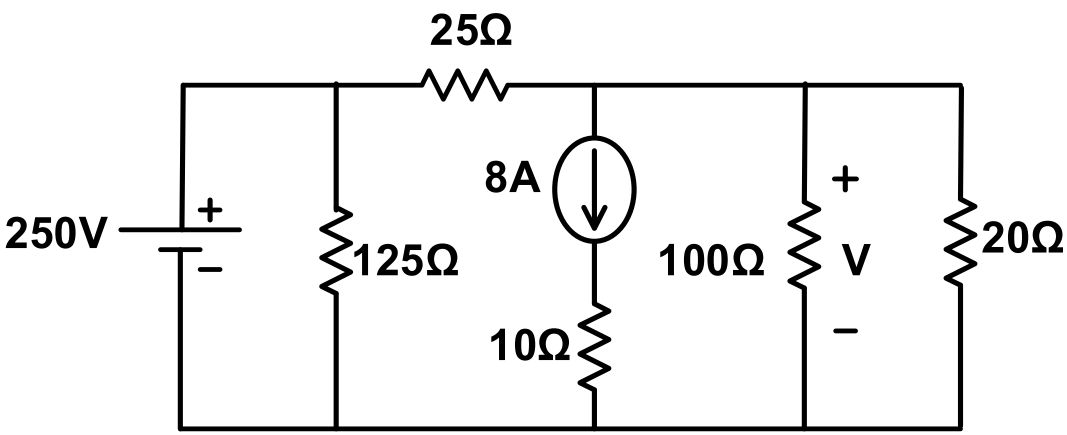


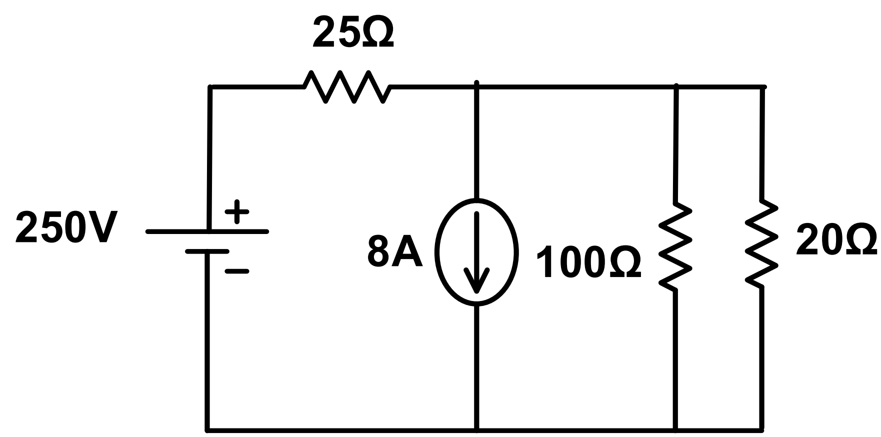
Example 10

Determine the voltage V

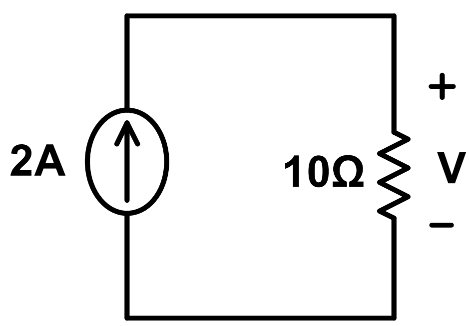


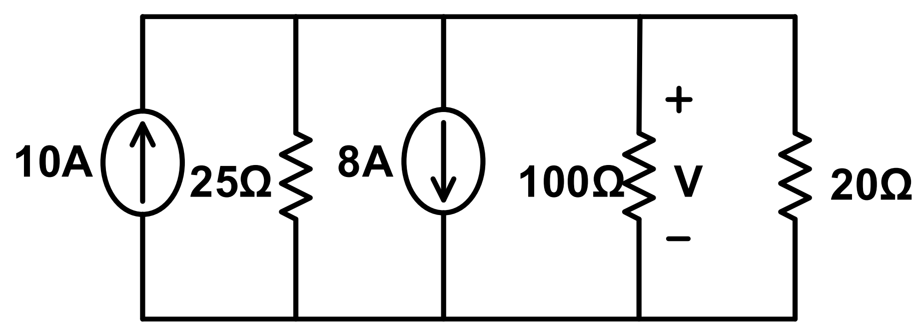
Solution of example 10





V = 2 X 10 = 20V





4. SUPERPOSITION METHOD

Principle

The superposition principle: For all linear systems, the net response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually => to test the linearity of a function.

Test the linearity

Is function $Y = 2X$ linear?

$X_1 = 1 \Rightarrow Y_1 = 2$

$X_2 = 2 \Rightarrow Y_2 = 4$

If $X_3 = X_1 + X_2 = 3$

Is $Y_3 = Y_1 + Y_2$ i.e., $Y_3 = 2 + 4 = 6$?

Test: $X_3 = 3 \Rightarrow Y_3 = 6$

Conclusion: This function is linear

Is $Y = 2X^2$ linear?

$X_1 = 1 \Rightarrow Y_1 = 2$

$X_2 = 2 \Rightarrow Y_2 = 8$

$X_3 = X_1 + X_2 = 3$

Is $Y_3 = Y_1 + Y_2$ i.e., $Y_3 = 2 + 8 = 10$?

Test: $X_3 = 3 \Rightarrow Y_3 = 18$

Conclusion: This function is nonlinear

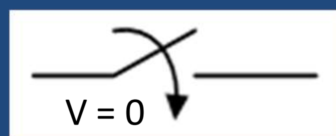
Superposition theorem

In an electrical circuit with many sources, the voltage or current is equal to the algebraic sum of the responses caused by each independent source acting alone.

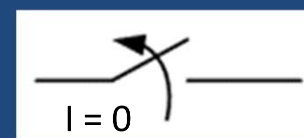
Method

1. Keep one source, kill other voltage and current sources

Kill voltage source



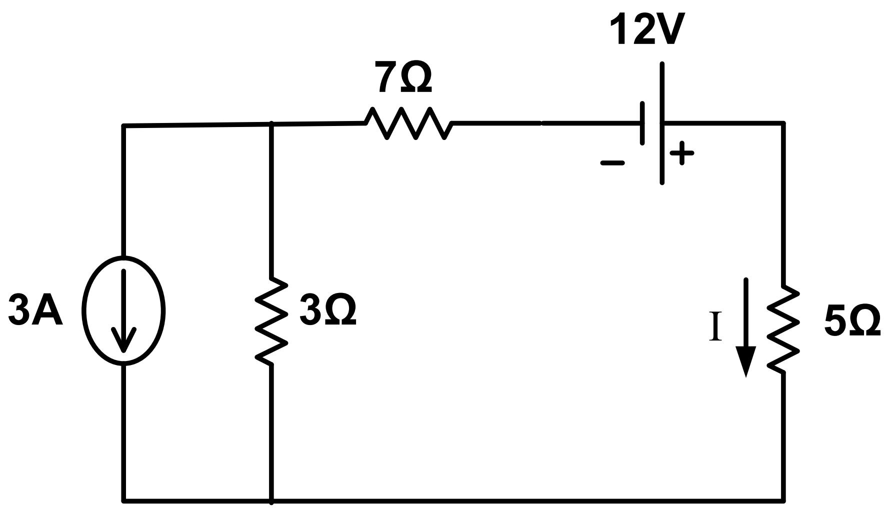
Kill current source

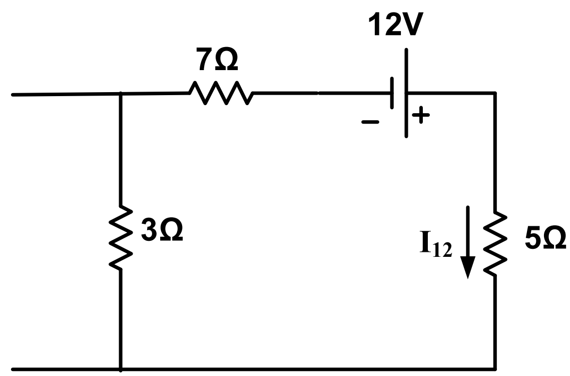


2. Calculate voltage or current due to the one remained source
3. Repeat the same with another source until the last source
4. Sum the results.

Warning: Apply to calculate only V or I ($V = RI$) but not $P = RI^2$

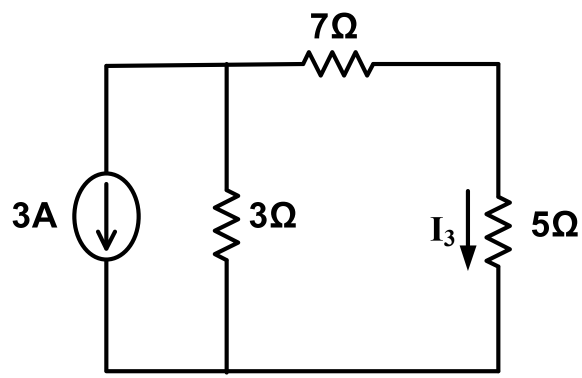
Example 11: Find I





Keep 12V
Kill 3A => OC

$$I_{12} = \frac{12}{7+3+5} = 0.8\text{A}$$



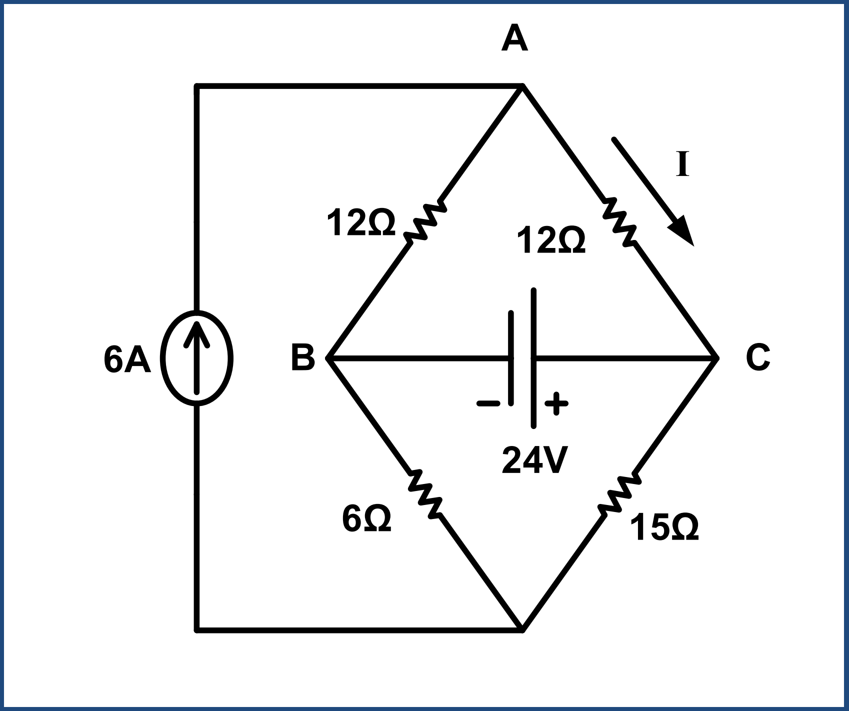
Keep 3A
Kill 12V => SC

$$I_3 = -3\,\frac{3}{7+3+5} = -0.6\text{A}$$

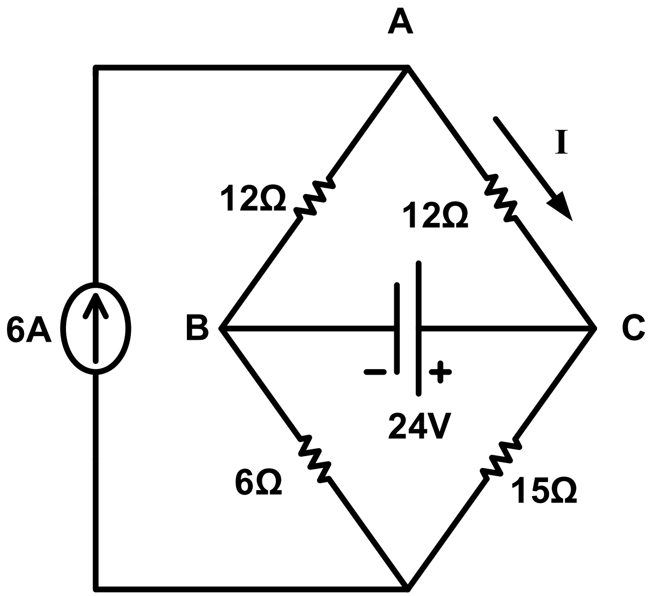
$$\begin{aligned} I &= I_{12} + I_3 \\ &= 0.8 - 0.6 = \underline{\mathbf{0.2A}} \end{aligned}$$

Example 12

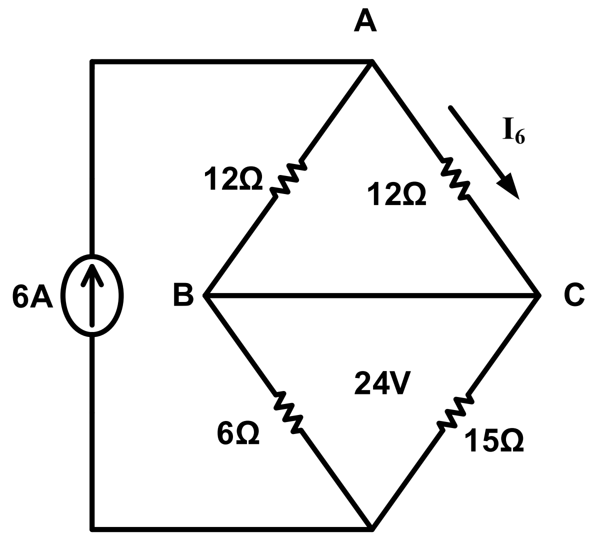
Determine the current I



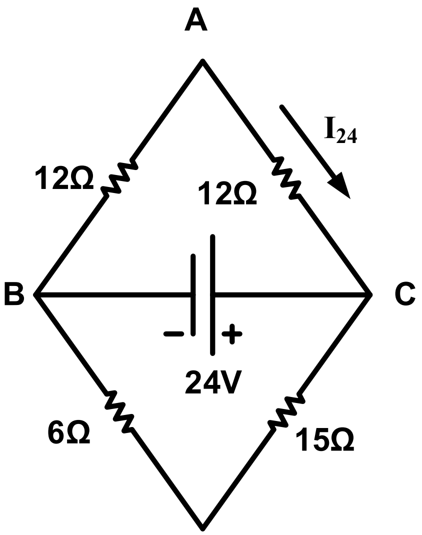
Solution of example 12



$$I = I_6 + I_{24}$$
$$= 3 - 1 = 2A$$

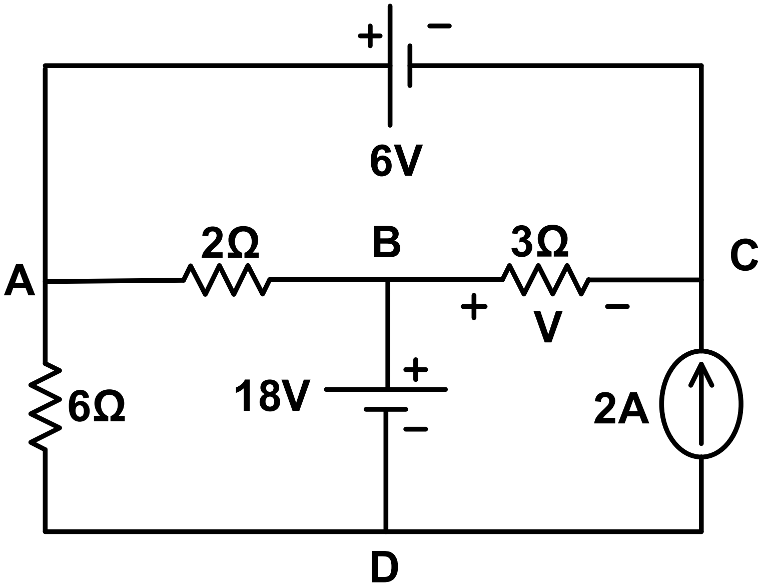


Keep 6A
Kill 24V => SC
Current divider:
$I_6 = 3A$

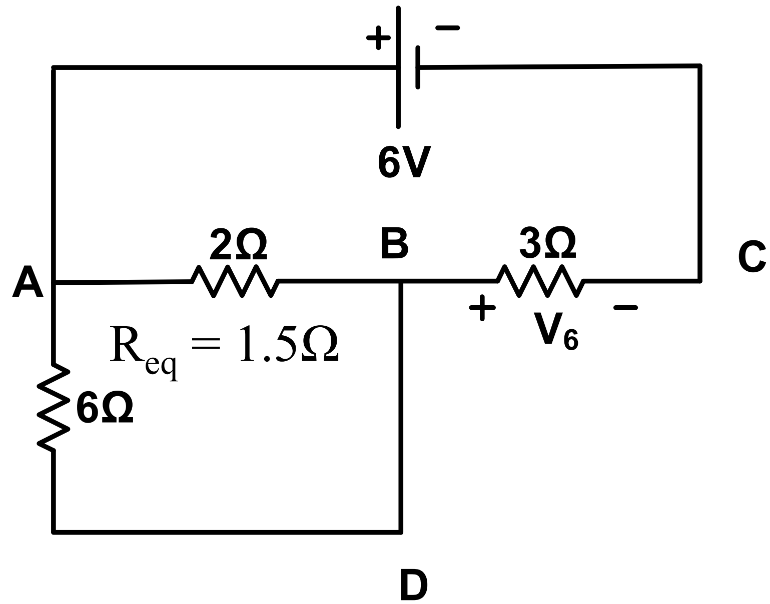


Keep 24V
Kill 6A => OC
Ohm's law
$I_{24} = - \ 24/(12+12) = -1A$

Solution of example 12 (cont.)



$$V = V_6 + V_{18} + V_2$$
$$= 4$$

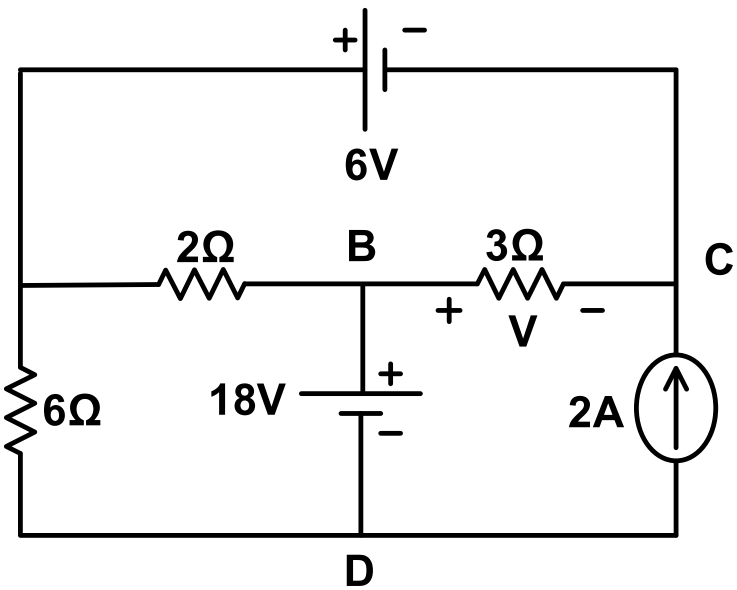


Keep 6V

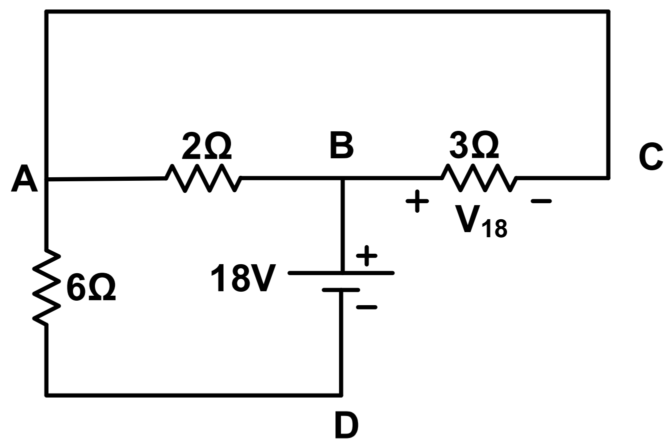
Kill 2A => OC; kill 18V => SC

Voltage divider $V_6 = 6\,\dfrac{3}{3 + 1.5} = 4V$

Solution of example 12 (cont.)



$$V = V_6 + V_{18} + V_2$$
$$= 4 + 3$$

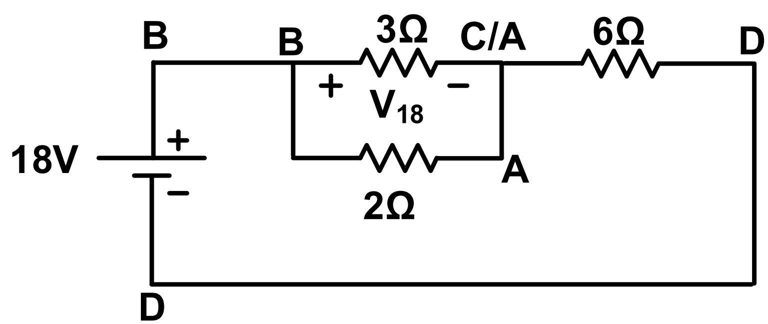


Keep 18V

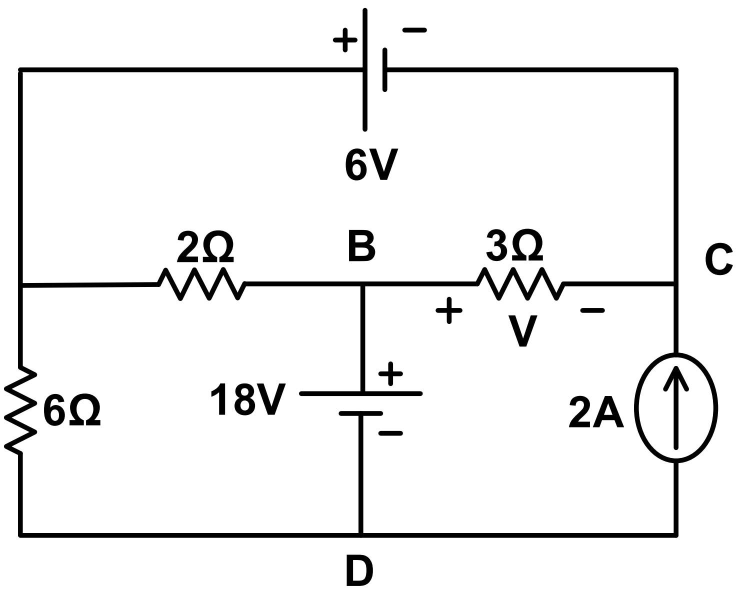
Kill 2A => OC; kill 6V => SC

Voltage divider:

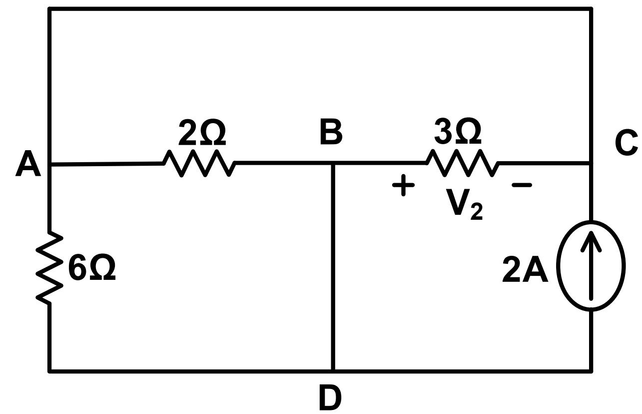
$$V_{18} = 18\,\frac{6/5}{(6/5)+6} = 3\text{V}$$

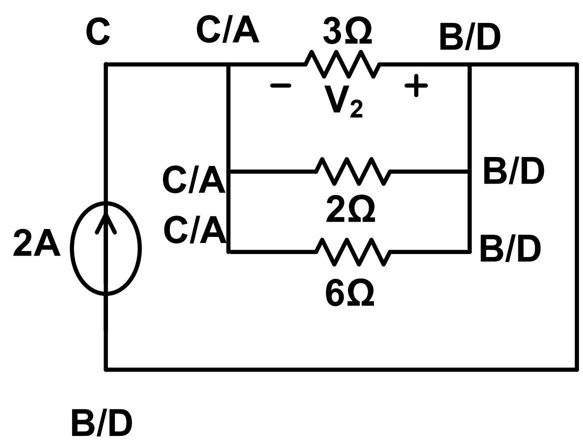


Solution of example 12 (cont.)



$$V = V_6 + V_{18} + V_2$$
$$= 4 + 3 - 2 = \underline{\mathbf{5V}}$$

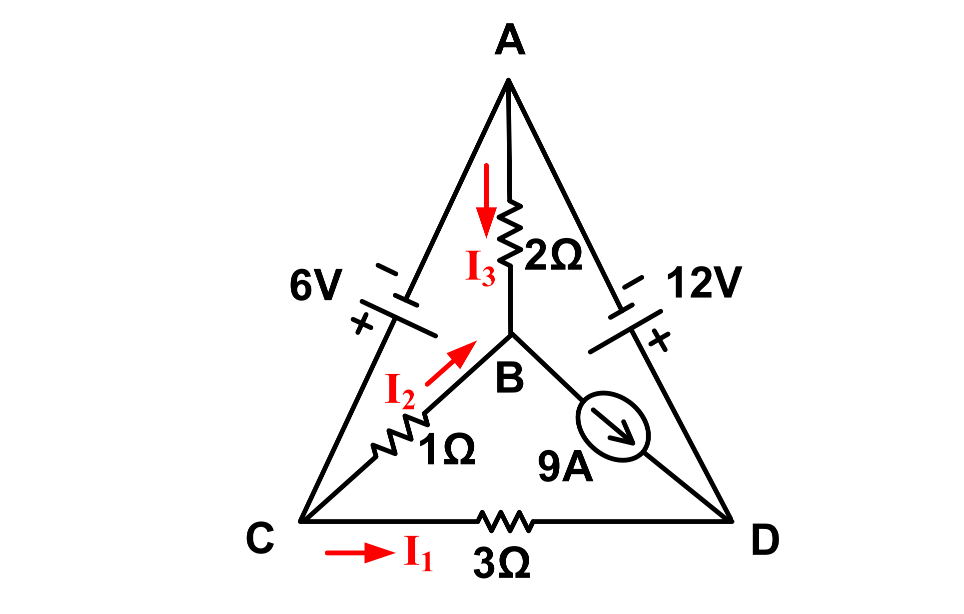




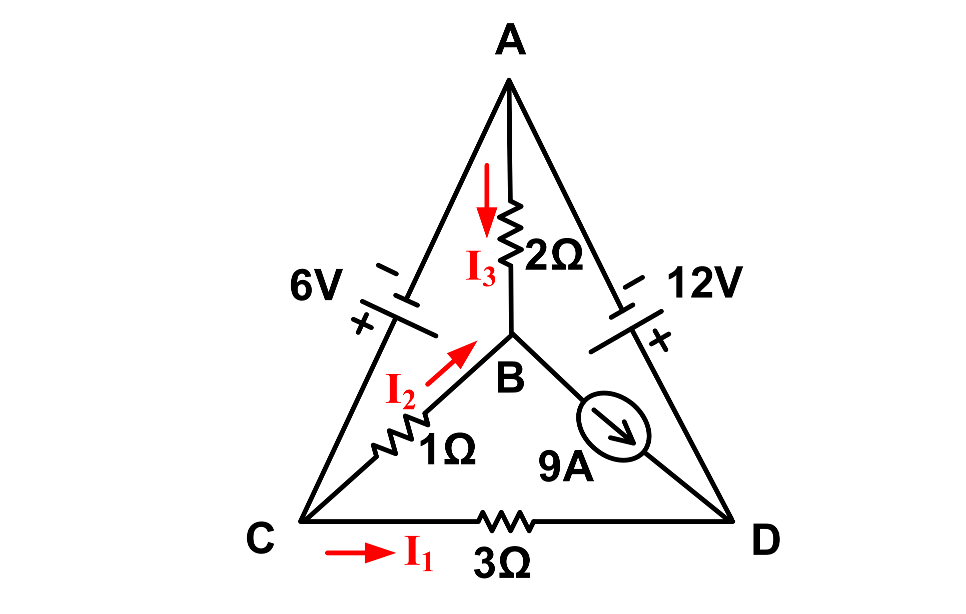
Keep 2A
Kill 6V => SC; kill 18V => SC
$R_{eq} = 1\Omega$
$V_2 = -2 \times 1 = -2$ V

Example 13

Determine the currents I_1, I_2 and I_3



Solution of example 13

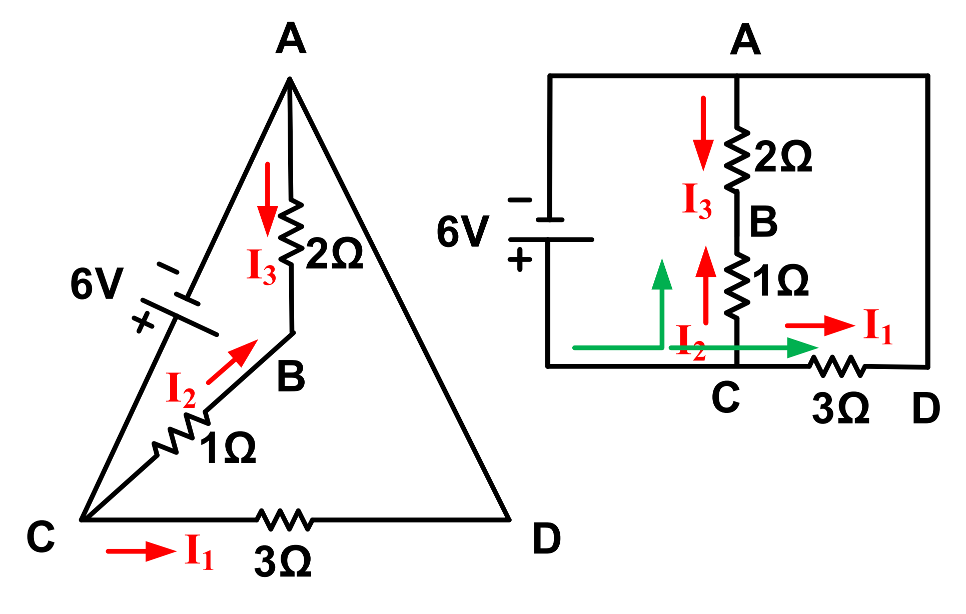


$I_1 = I_{1.6} + I_{1.12} + I_{1.9}$

$I_2 = I_{2.6} + I_{2.12} + I_{2.9}$

$I_3 = I_{3.6} + I_{3.12} + I_{3.9}$

Solution of example 13 (cont.)



$I_1 = I_{1.6} + I_{1.12} + I_{1.9}$

$\quad = 2 + \ldots$

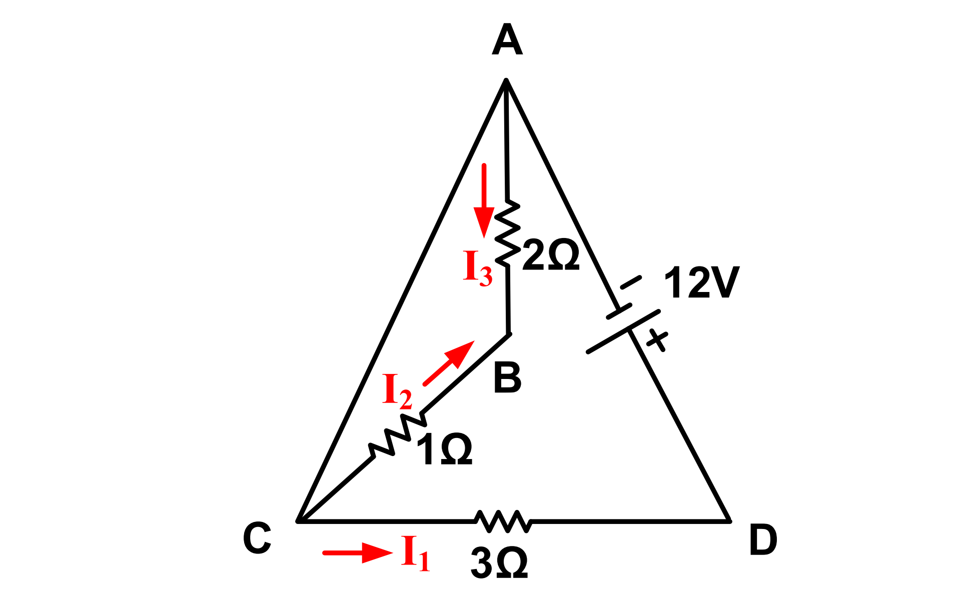
$I_2 = I_{2.6} + I_{2.12} + I_{2.9}$

$\quad = 2 + \ldots$

$I_3 = I_{3.6} + I_{3.12} + I_{3.9}$

$\quad = -2 + \ldots$

Solution of example 13 (cont.)

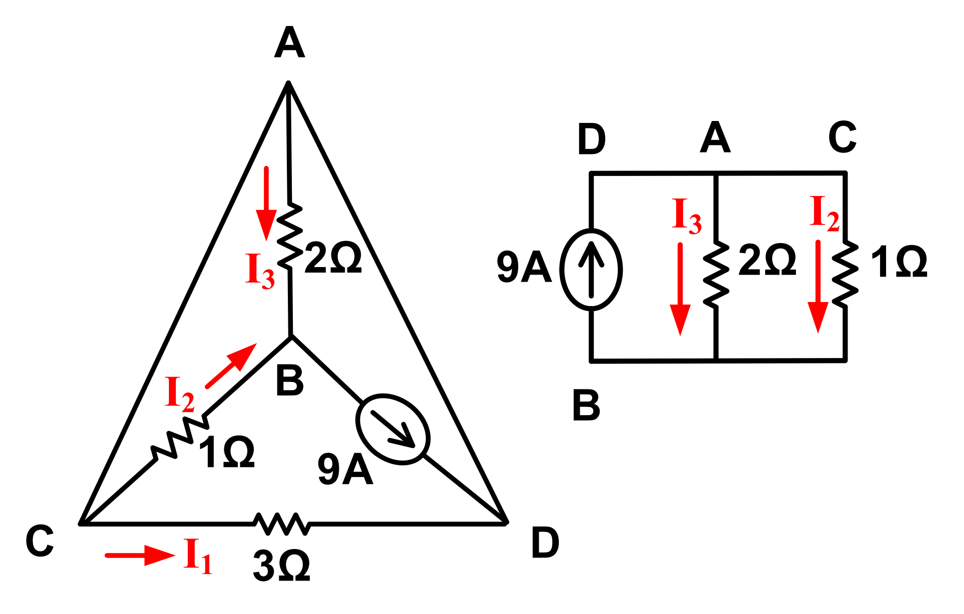


$$I_1 = I_{1.6} + I_{1.12} + I_{1.9}$$
$$= 2 - 4 + \dots$$

$$I_2 = I_{2.6} + I_{2.12} + I_{2.9}$$
$$= 2 + 0 + \dots$$

$$I_3 = I_{3.6} + I_{3.12} + I_{3.9}$$
$$= -2 + 0 + \dots$$

Solution of example 13 (cont.)



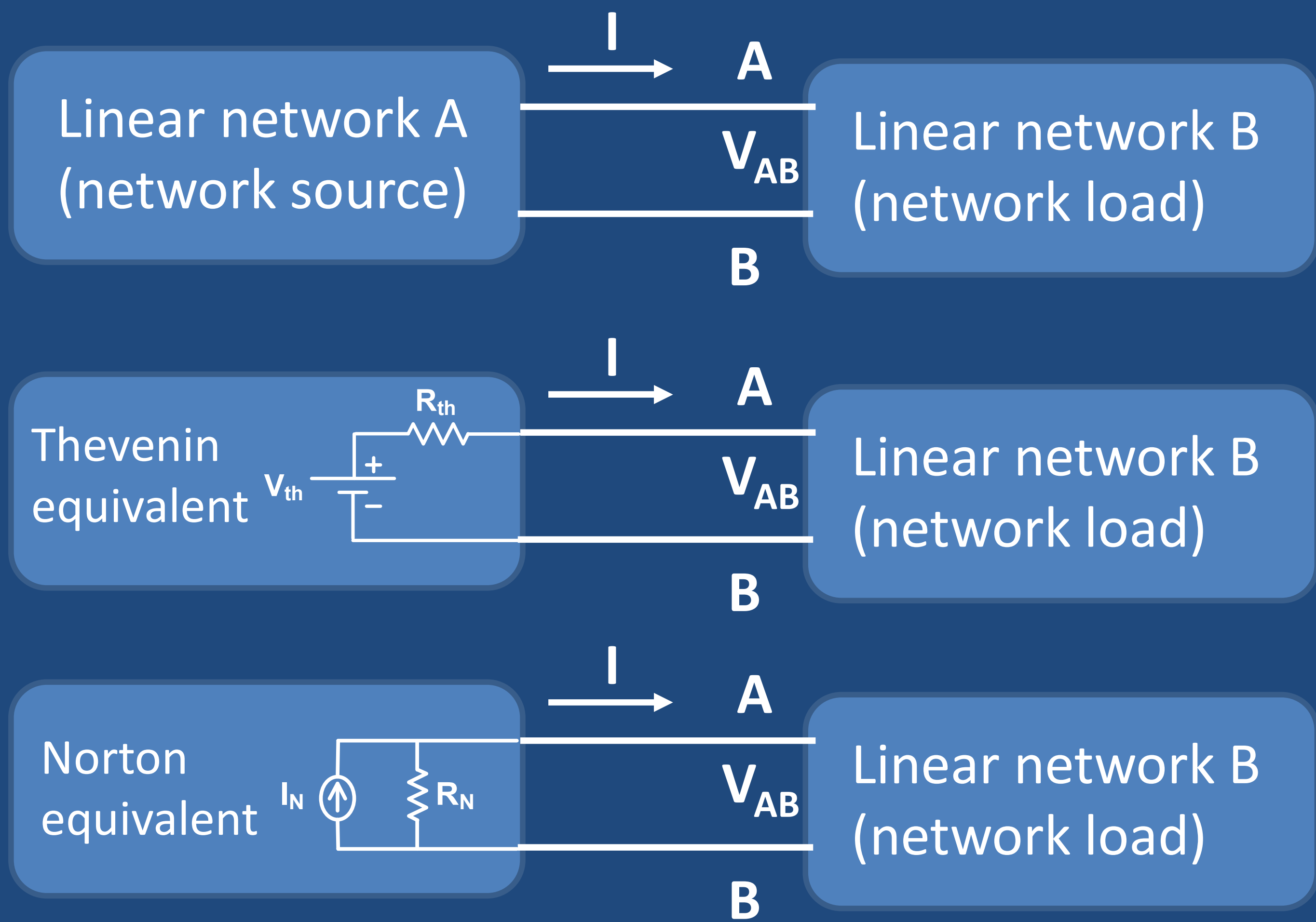
$$I_1 = I_{1.6} + I_{1.12} + I_{1.9}$$
$$= 2 - 4 + 0 = -2A$$

$$I_2 = I_{2.6} + I_{2.12} + I_{2.9}$$
$$= 2 + 0 + 6 = 8A$$

$$I_3 = I_{3.6} + I_{3.12} + I_{3.9}$$
$$= -2 + 0 + 3 = 1A$$

5. THEVENIN AND NORTON METHODS

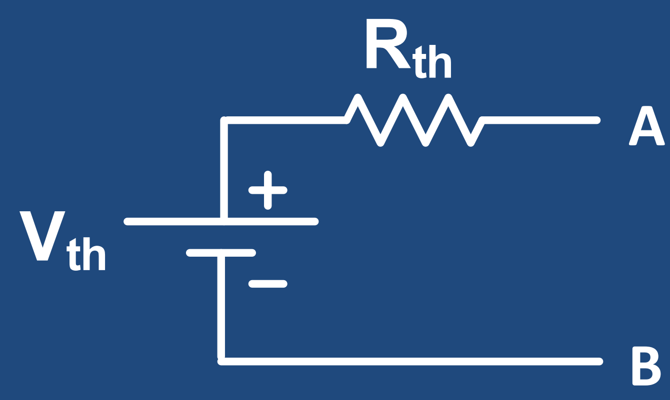
Concept of networks



Thevenin equivalent circuit

Statement:

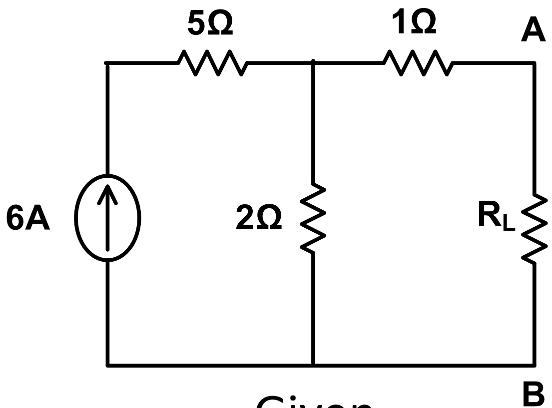
1. $V_{th} = V_{AB|oc}$
2. R_{th} = Resistance of the dead network



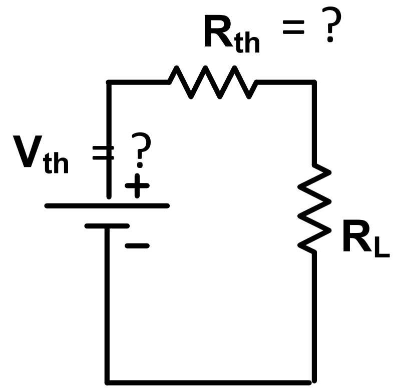
Method:

1. Remove the load
2. Kill all sources then calculate R_{eq} looked from A and B => R_{th}
3. Put back all sources then calculate $V_{AB|oc}$ => V_{th}

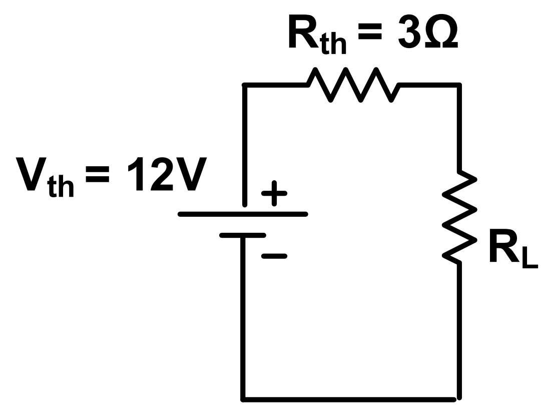
Example 14: Determine the Thevenin equivalent circuit



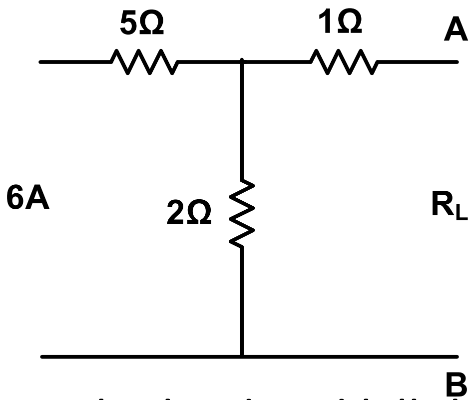
Given



Questions

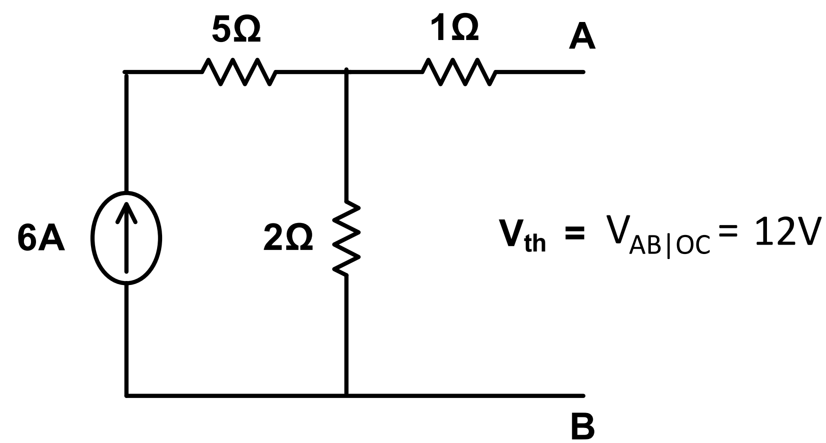


Results



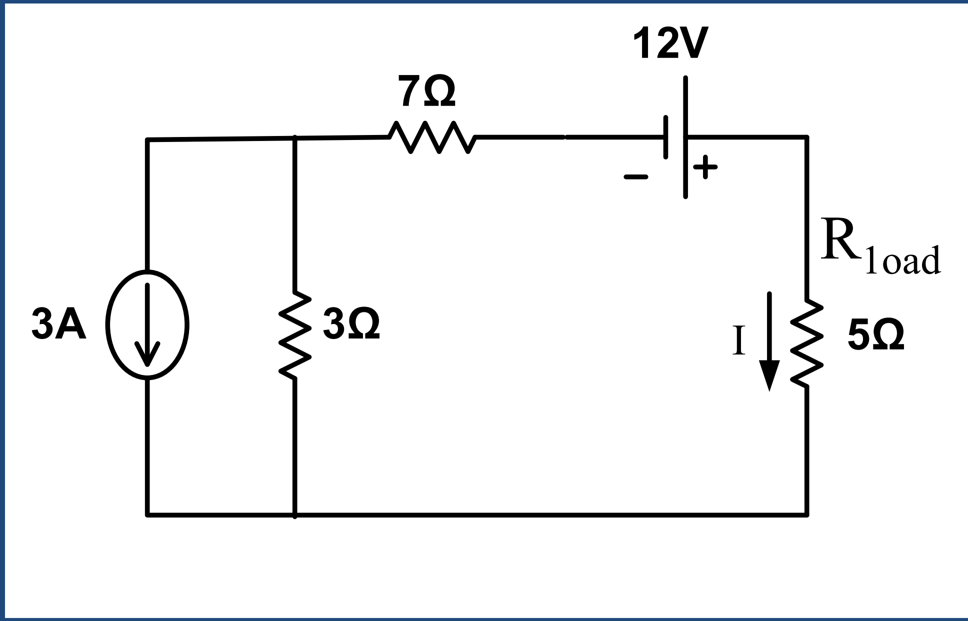
Remove the load and kill the source:

$R_{th} = R_{AB|OC} = 3\Omega$

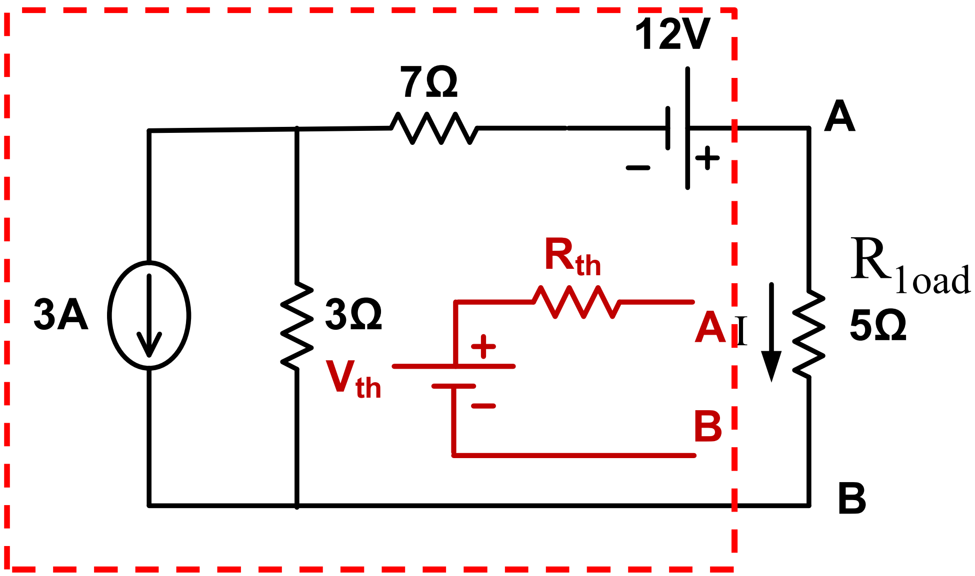


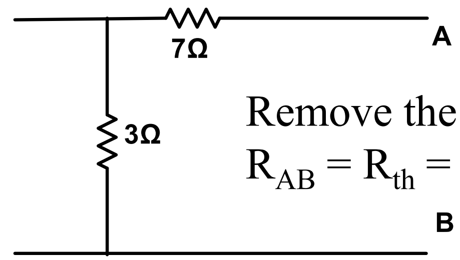
Example 15

Determine the current I



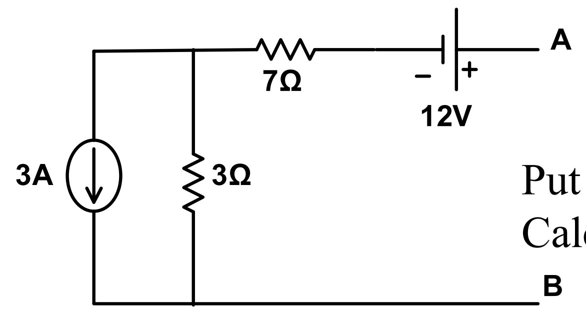
Solution of example 15





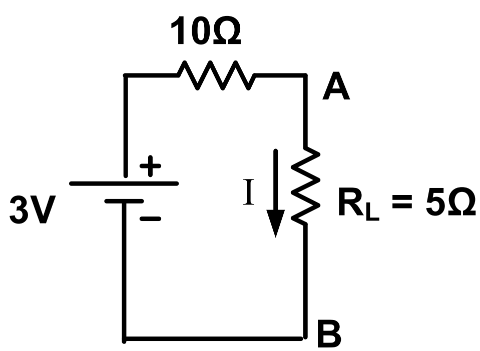
Remove the load and kill all sources

$R_{AB} = R_{th} = 7 + 3 = 10\Omega$

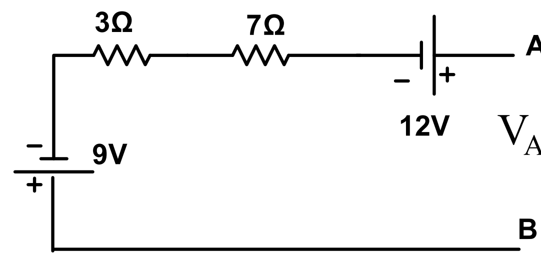


Put back all sources

Calculate V_{AB}



$$I = \frac{3}{10+5} = \underline{\mathbf{0.2A}}$$

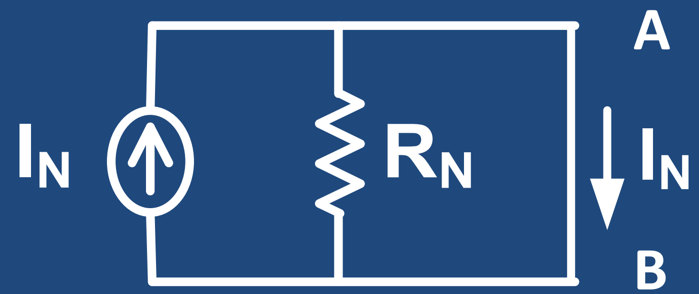


$V_{AB} = V_{th} = 12 - 9 = 3V$

Norton equivalent circuit

Statement:

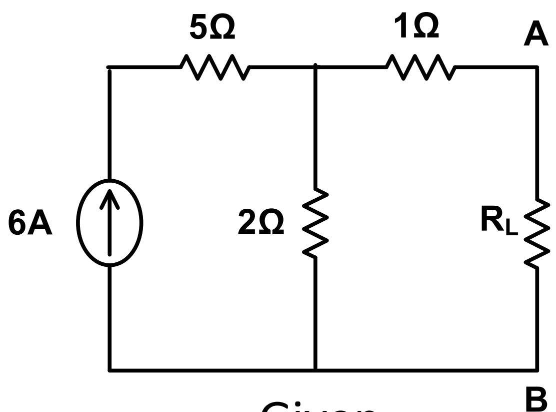
1. R_N = Resistance of the dead network
2. $I_N = I_{AB|sc}$



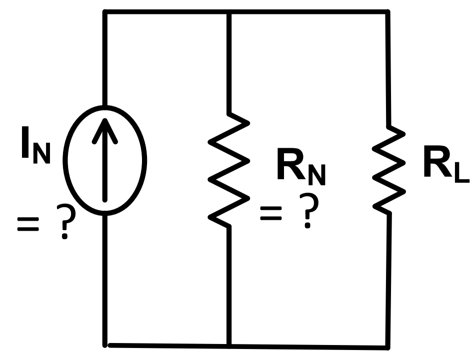
Method:

1. Remove the load
2. Kill all sources then calculate R_{eq} looked from A and B => R_N
3. Put back all sources, short circuit AB then calculate $I_{AB|sc}$ => I_N

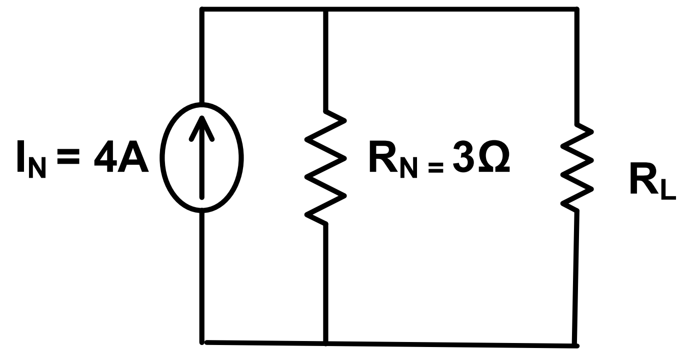
Example 16: Determine the Norton equivalent circuit



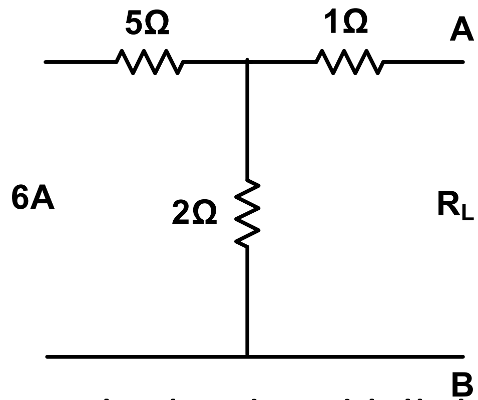
Given



Questions

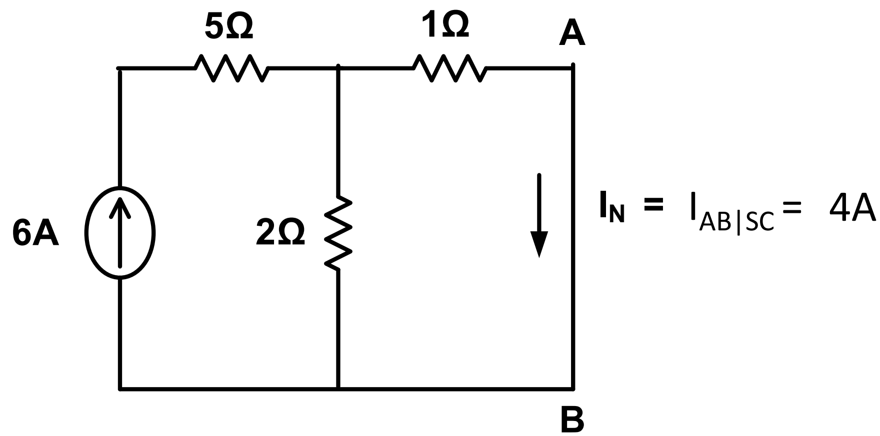


Results



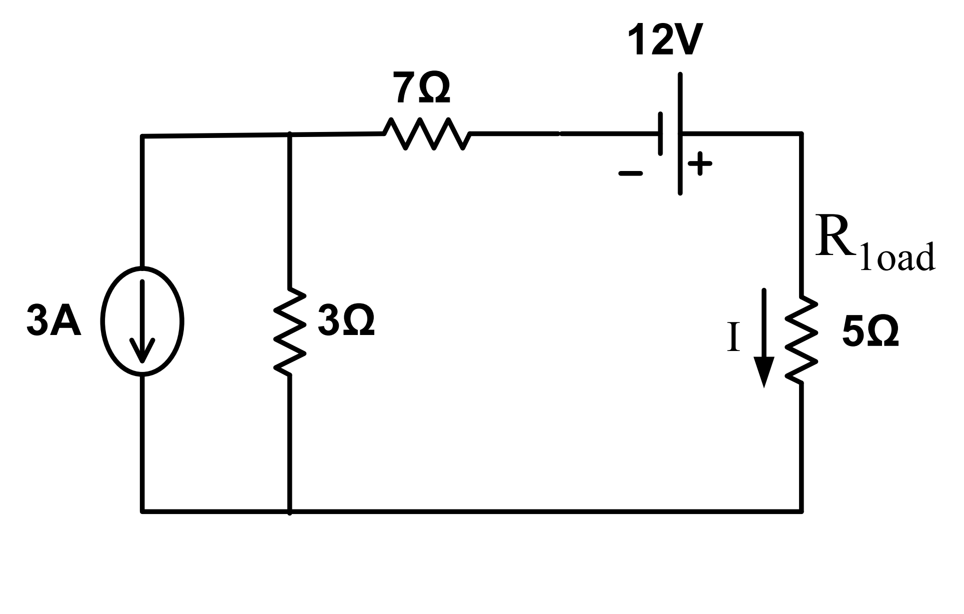
Remove the load and kill the source:

$R_{th} = R_{AB|OC} = 3\Omega$

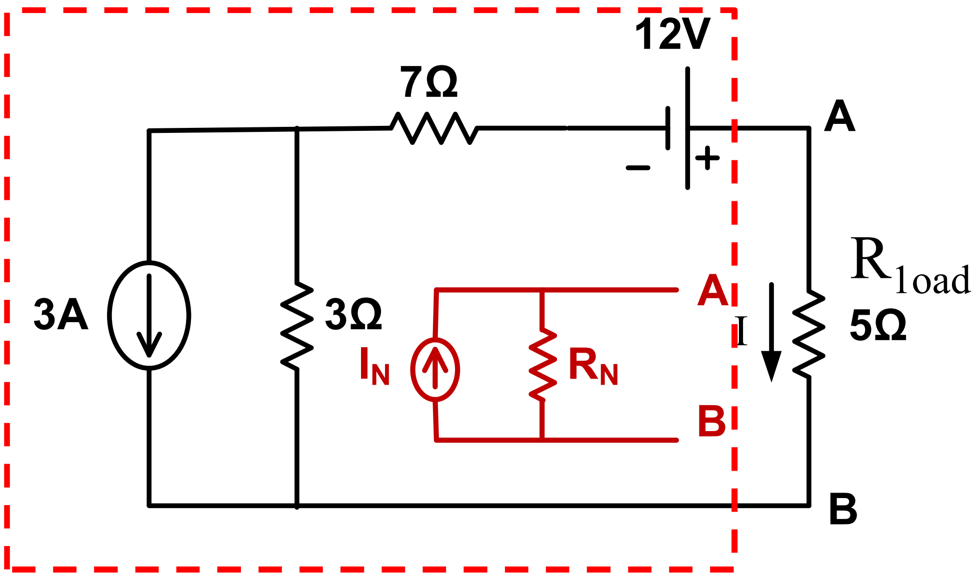


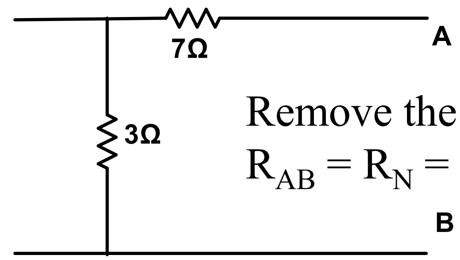
Example 17

Determine the current I



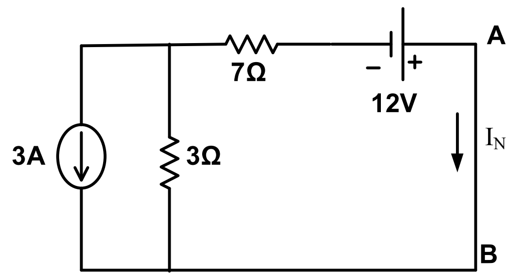
Solution of example 17





Remove the load and kill all sources

$R_{AB} = R_N = 7 + 3 = 10\Omega$

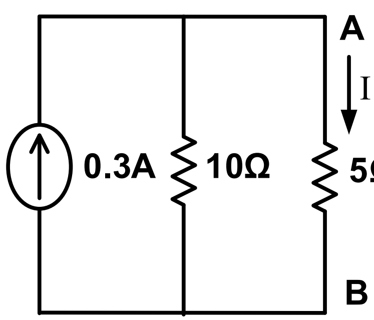


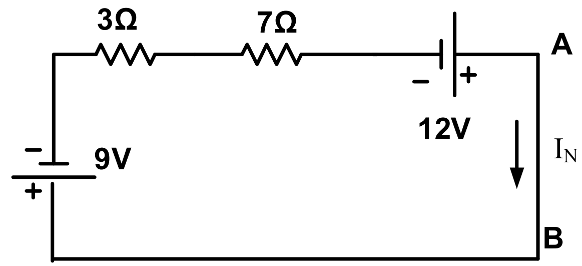
Put back all sources

SC AB

Calculate I_{AB}

$$I = 0.3\,\frac{10}{10+5} = \underline{\mathbf{0.2A}}$$

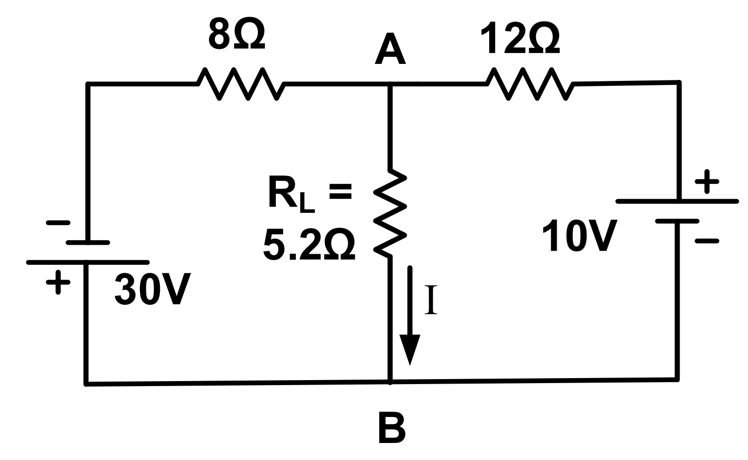




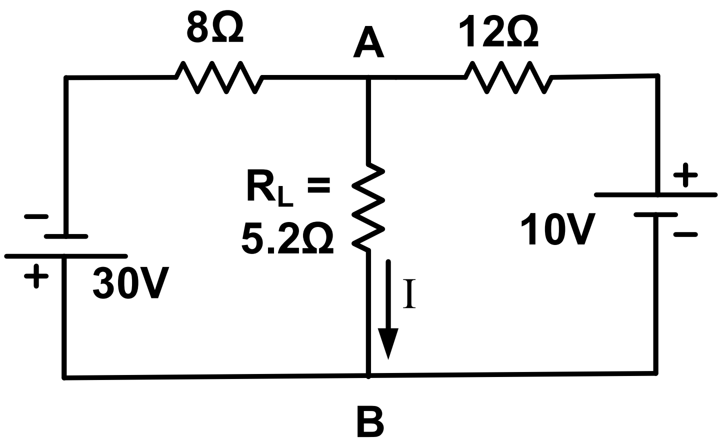
$$I_{AB} = I_N = \frac{12-9}{3+7} = 0.3A$$

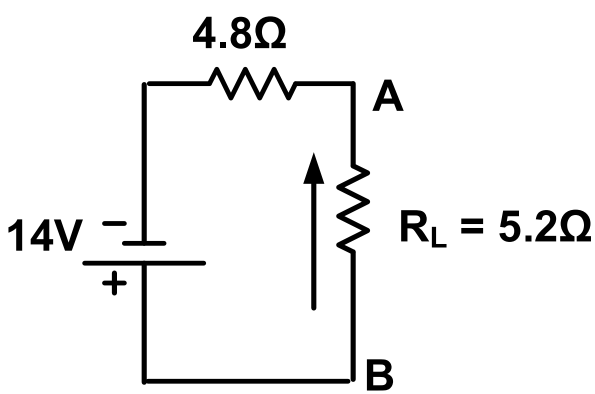
Example 18

Determine the current I



Solution of example 18: Thevenin equivalent





Calculate R_{th}

$$R_{th} = \frac{8 * 12}{8 + 12} = 4.8\Omega$$

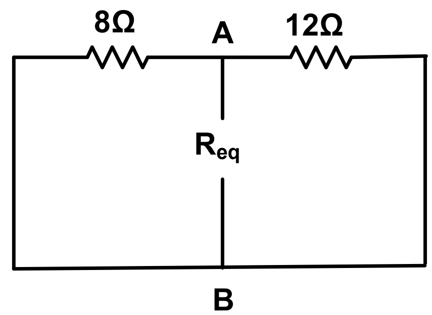
Calculate V_{th}

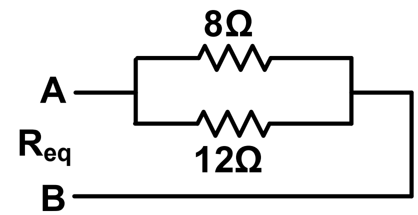
$$V_{th} = V_{AB} = V_{AC} + V_{CB}$$

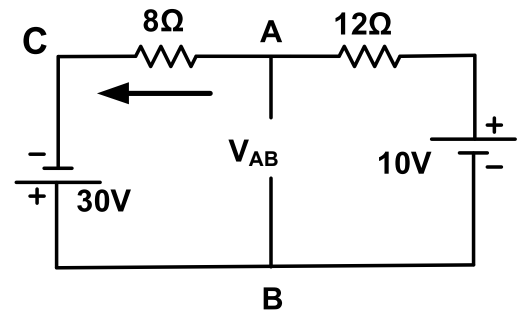
$$V_{AC} = \frac{30+}{8+12}\, 8 = 16V$$

$$V_{th} = V_{AB} = 16 - 30 = -14V$$

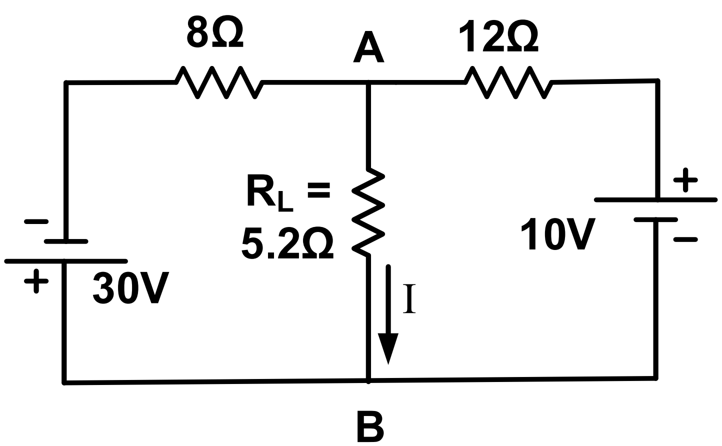
$$I = \frac{-14}{4.8 + 5.2} = -\underline{\mathbf{1.4A}}$$





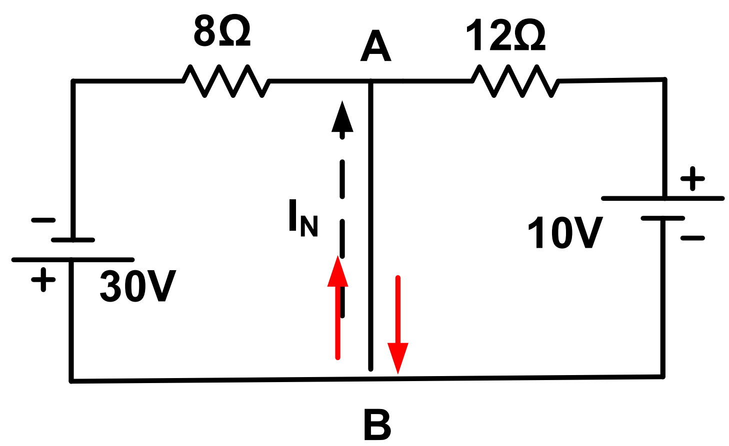


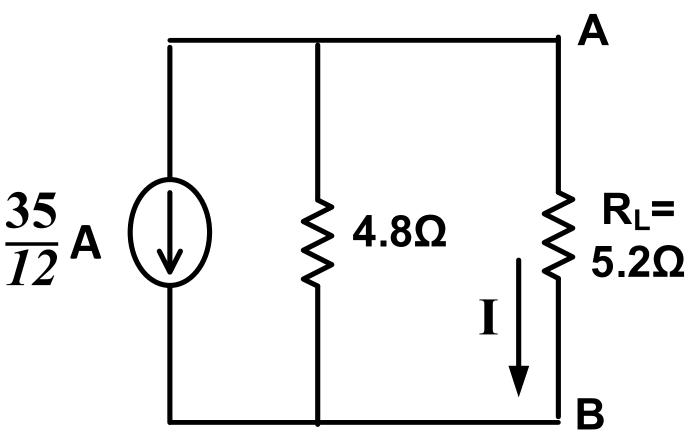
Solution of example 18: Norton equivalent



$R_N = R_{th} =$
4.8Ω
Calculate I_N

$$I_N = \frac{30}{8} - \frac{10}{12} = \frac{35}{12}\text{A}$$

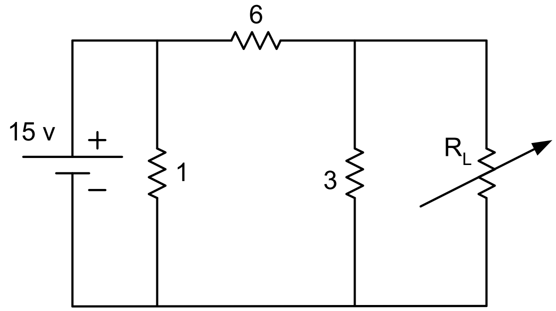




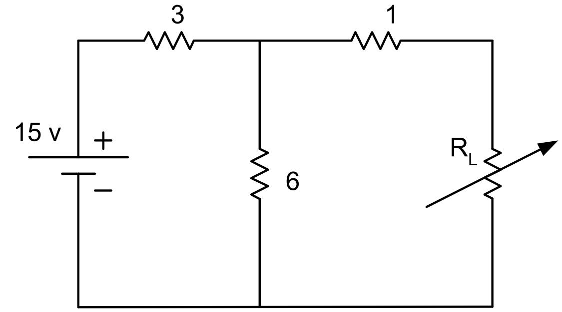
$$I = -\frac{35}{12}\frac{4.8}{4.8+5.2} = -\ 1.4\text{A}$$

Example 19:

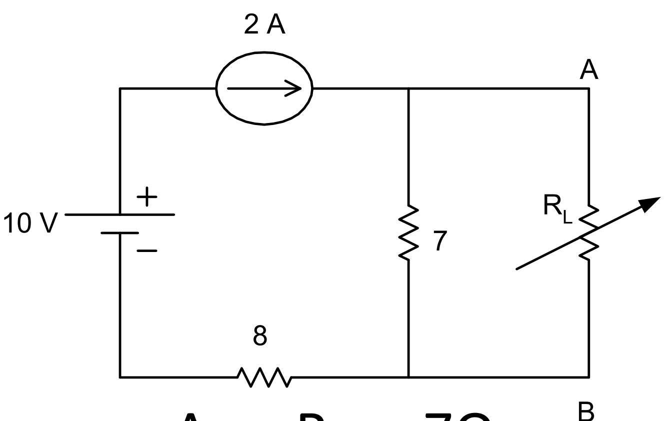
Find the Thevenin equivalent circuits of the following ones



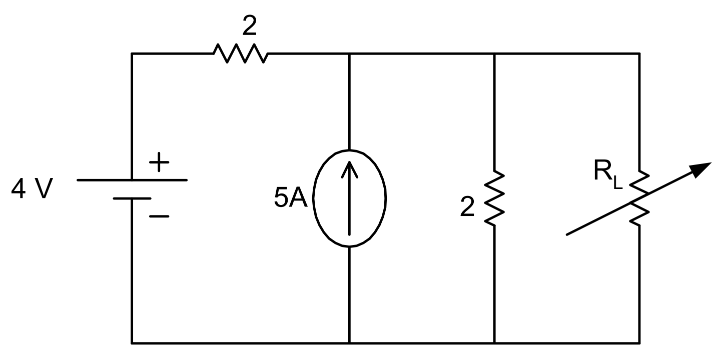
Ans: $R_{th} = 2\Omega$
$V_{th} = 5V$



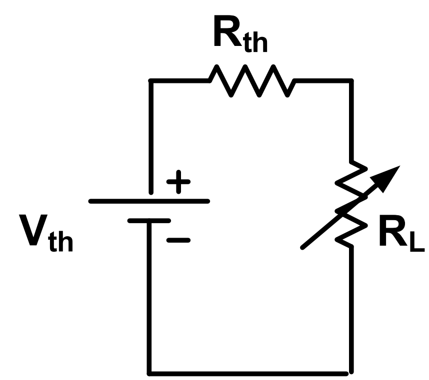
Ans: $R_{th} = 3\Omega$
$V_{th} = 10V$



Ans: $R_{th} = 7\Omega$
$V_{th} = 14V$

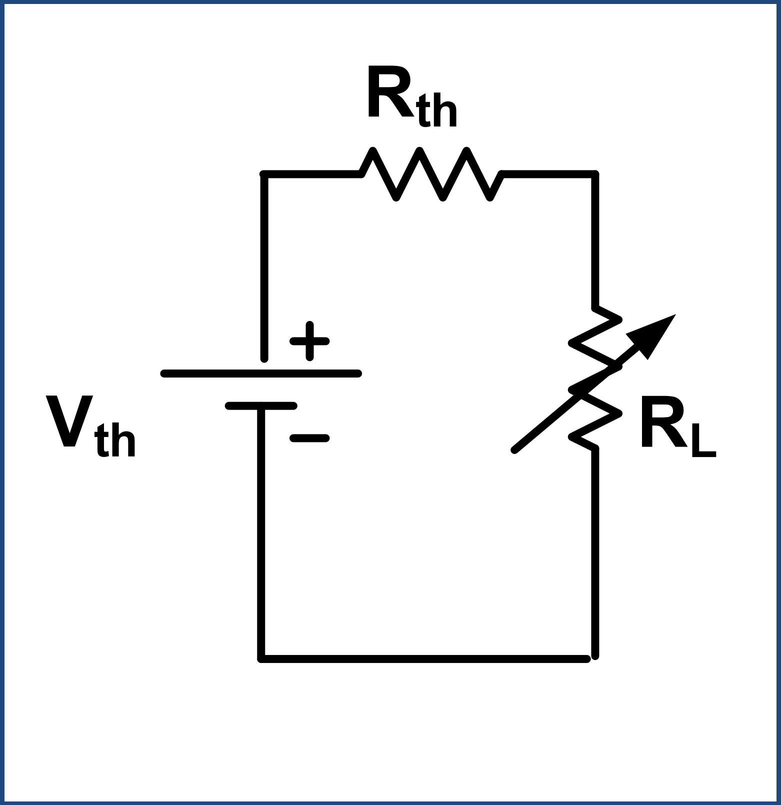


Ans: $R_{th} = 1\Omega$
$V_{th} = 7V$



Maximum power transfer theorem (when R_L varies)

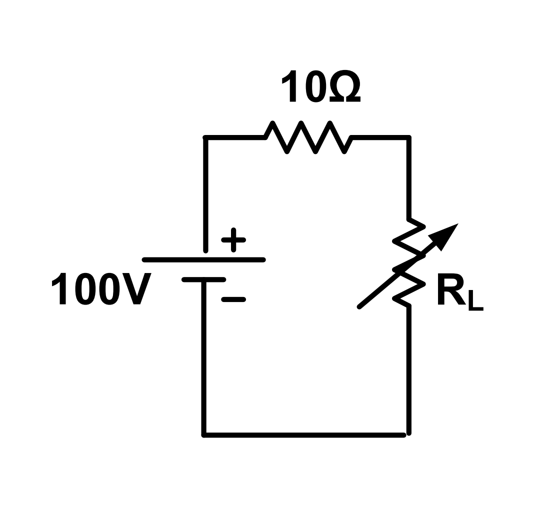
1. When $R_L = R_{th}$
2. $P_{Lmax} = \frac{V_{th}^2}{4R_{th}}$



Power transfer efficiency

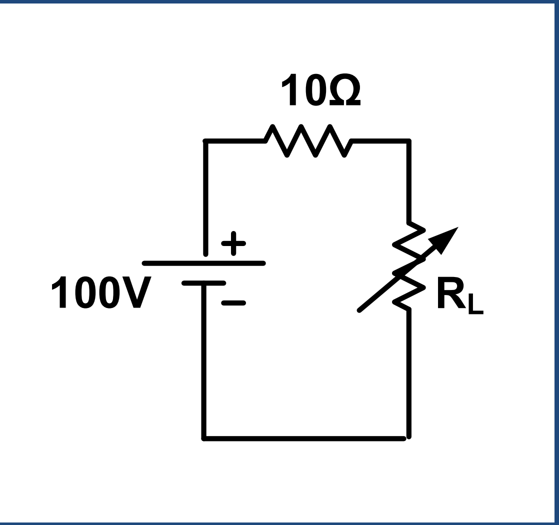
$$\eta = \frac{P_{out}}{P_{in}} = \frac{R_L}{R_L + R_{th}}$$

Example 20



1. Find P_{Lmax}
2. If P_L = 160W find R_L and η

Solution of example 20



- $P_{Lmax} = \frac{100^2}{4*10} = 250$ W
- $P_L = R_L I^2$
- $\quad = R_L \left(\frac{100}{10 \quad R_L}\right)^2 = 160$

- or $R_L^2 - 42.5 R_L + 100 = 0$
- => Solutions: $R_L = 40\Omega$ and 2.5Ω
- $\eta = = \frac{R_L}{R_L + Rth}$
  - For $R_L = 40\Omega \Rightarrow \eta_1 = \frac{40}{40 + 10} 100 = 80\%$
  - For $R_L = 2.5\Omega \Rightarrow \eta_2 = \frac{2.5}{2.5 + 10} 100 = 20\%$

- => Take $R_L = 40\Omega$ to have $\eta = 80\%$ for the same power.

Võ Văn Tới
School of Biomedical Engineering
International University of Vietnam National Universities
HCM City, Vietnam
Email: vvtoi@hcmiu.edu.vn
Website: www.hcmiu.edu.vn/bme