Principle of EE1
Lesson 6

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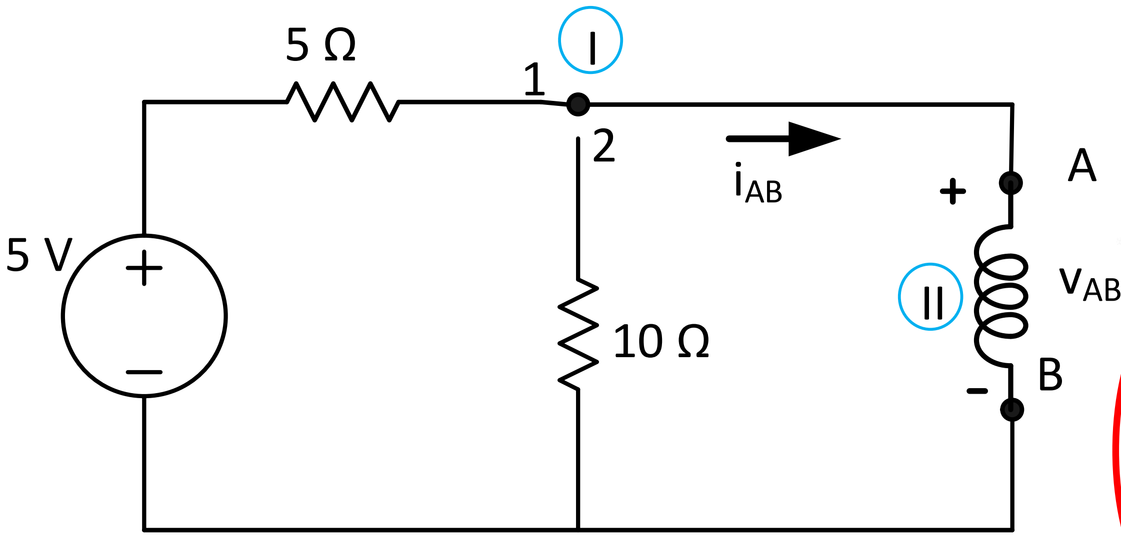
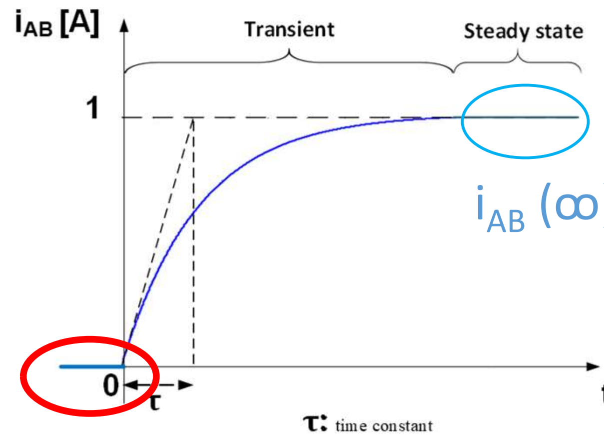
Inductor and Inductance

Phenomenon

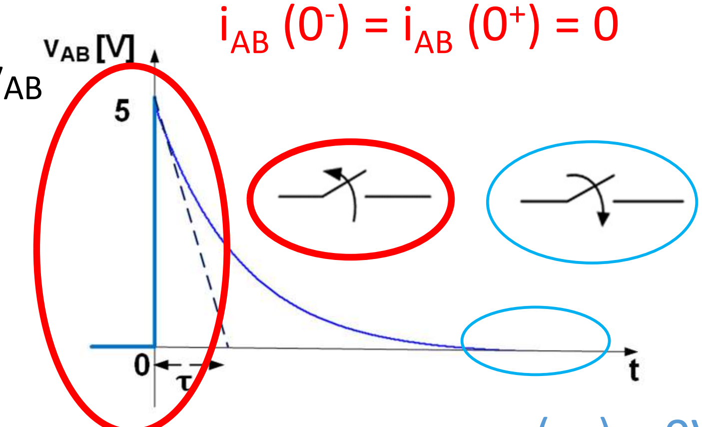
Inductor is charging





i_{AB} (∞) = 1A

i_{AB} (0^-) = i_{AB} (0^+) = 0



v_{AB} (∞) = 0V

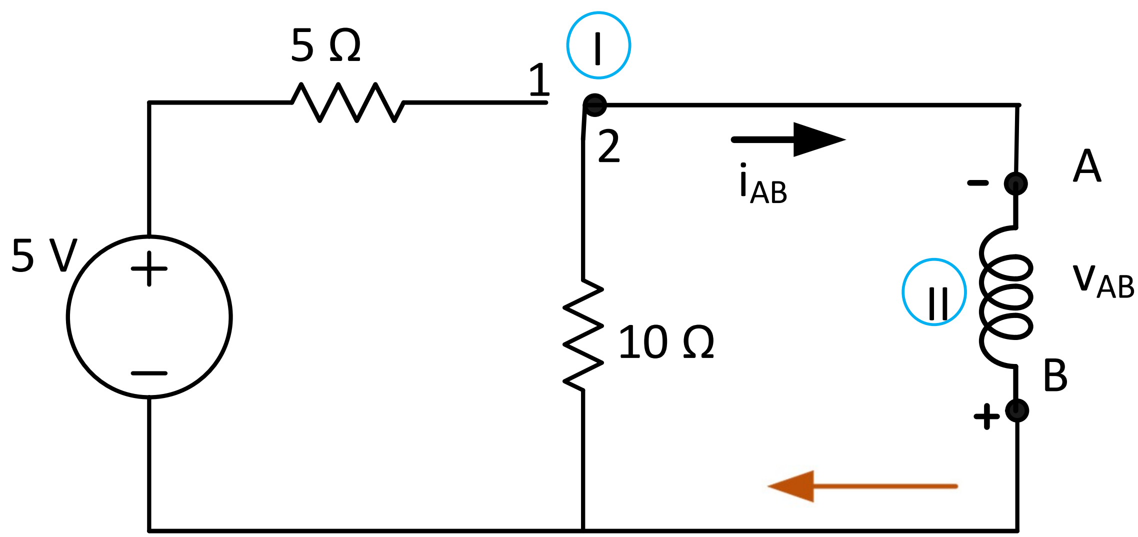
v_{AB} (0^-) = 0V

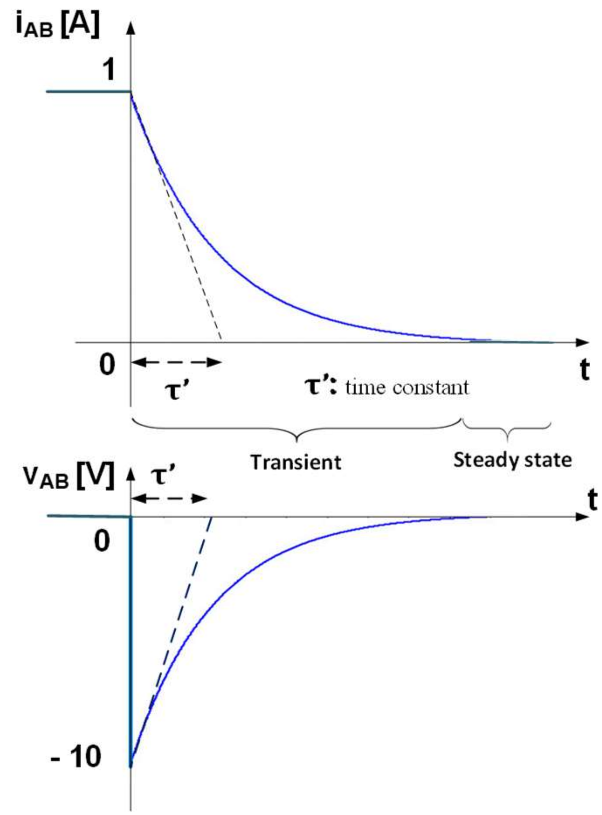
v_{AB} (0+) = 5V

II is called an Inductor = storing energy element. It can be empty or full.

Empty => Open switch, Full => Closed switch

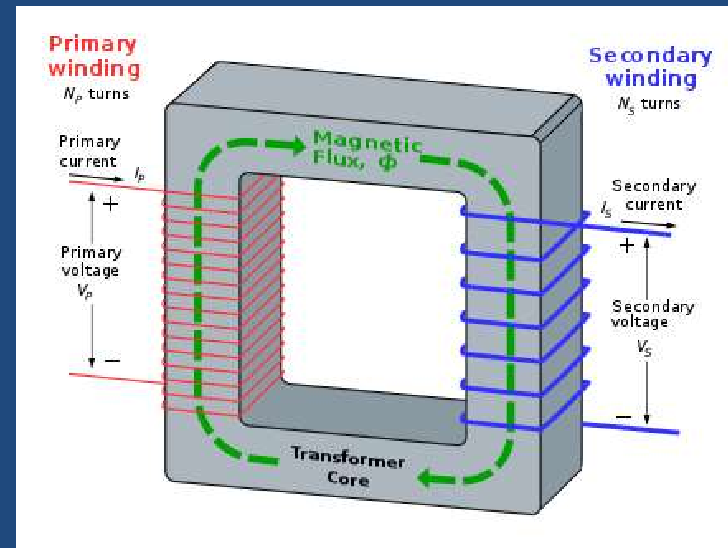
Inductor is discharging, it acts like an energy source



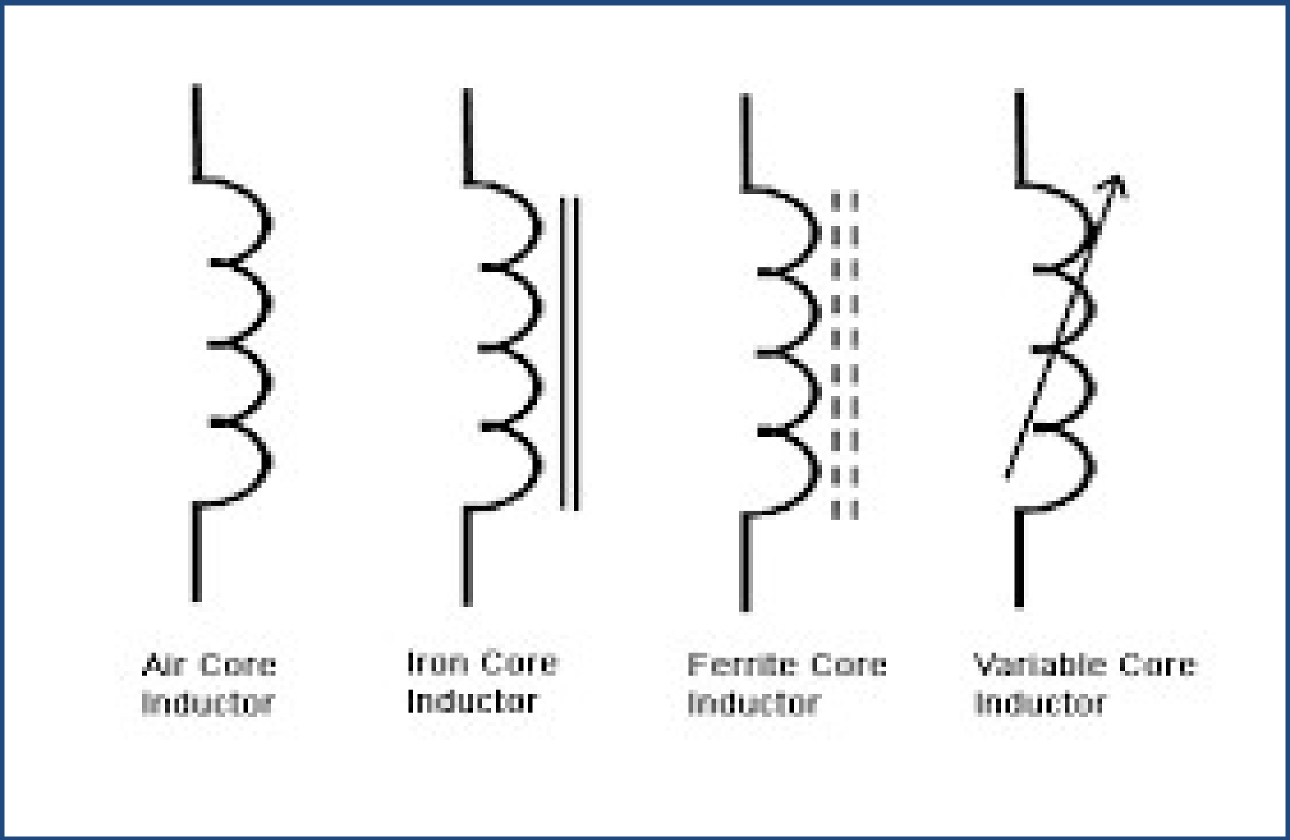


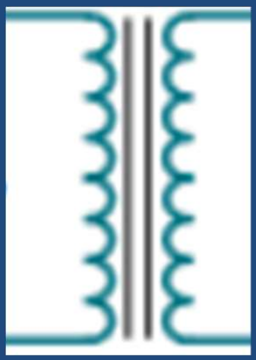
Inductors

Transformer



Symbols of inductor





Transformer

Inductance

$$L = \frac{\mu N^2 A}{l}$$

Where:

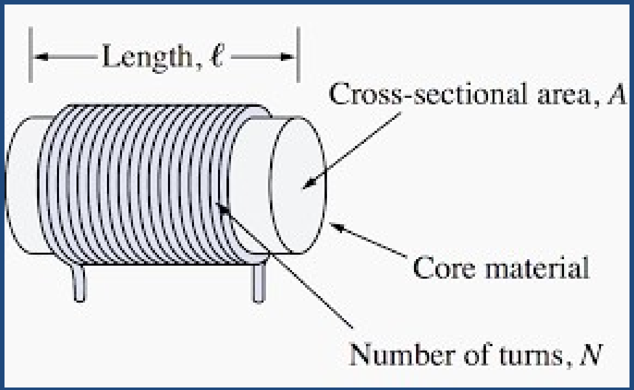
$L = \mathit{Inductance\ in\ henries}\ (H)$

$\mu = \mathit{permeability}\ (Wb/A \cdot m)$

$N = \mathit{number\ of\ turns\ in\ coil}$

$A = \mathit{area\ encircled\ by\ coil}\ (m^2)$

$l = \mathit{lenth\ of\ coil}\ (m)$



Basic notes

1. Inductor is a storing energy element: it can be charged and discharged => time constant $\tau = L/R_{th}$
2. Empty => Open switch, Full => Closed switch
3. Magnetic flux φ [Wb] = L [H] . I [A]
4. Stored energy W [J] = ½ L [H] . I^2 [A]
5. Current cannot change instantly $i(0^+) = i(0^-)$
6. Voltage changes instantly v(0) = max
7. $v_L(t) = \frac{d\varphi}{dt} = L\,\frac{di}{dt}$

Inductor connections

- **Series**
  - Same current
  - $L_{eq} = L_1 + L_2 + L_3 + \ldots$

- **Parallel**
  - Same voltage V
  - $1/L_{eq} = 1/L_1 + 1/L_2 + 1/L_3 + \ldots$

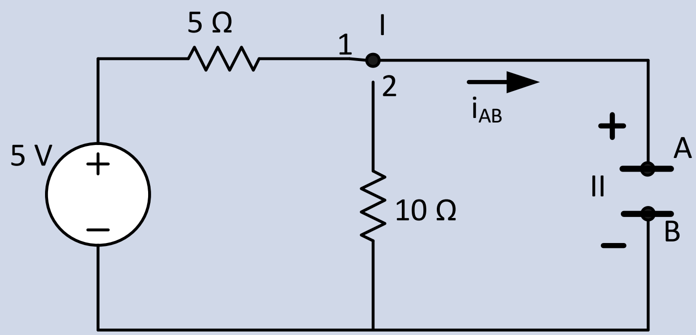
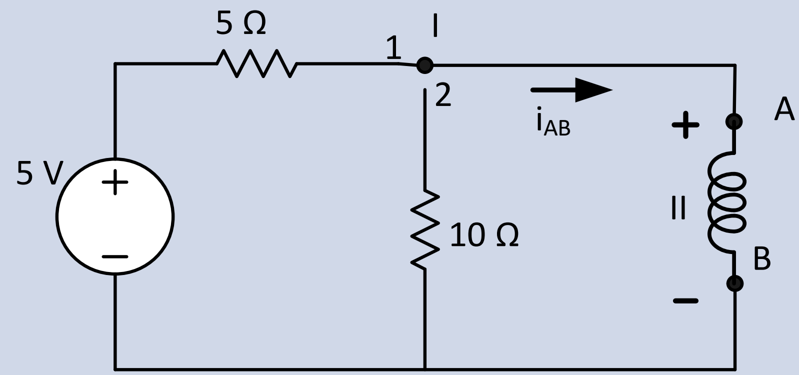
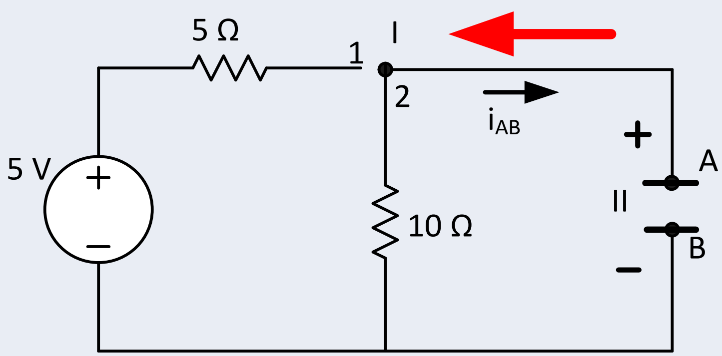
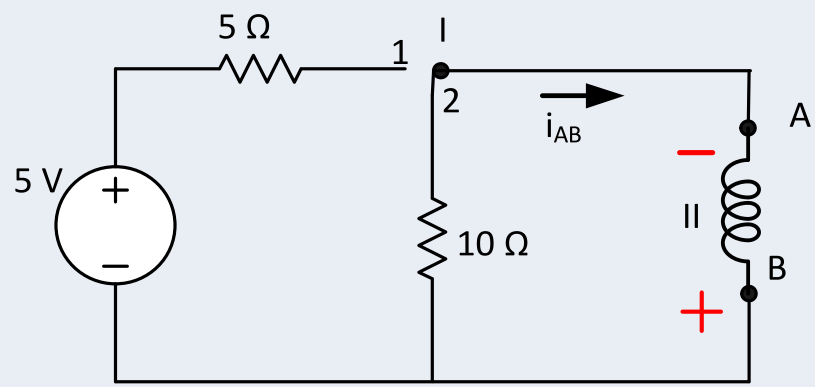
General equations

$$i_L(t) = I_{ss} + k\, e^{-\frac{t}{\tau}}$$

- $i_L(t)$: *instantaneous current valid for all t*
- I_{ss}*: steady state current i.e. when* $i_L(\infty)$
- k*: constant =* $I_L(0)$ - I_{ss}
- $\tau = L/R_{th}$

$$v_L(t) = \frac{d\varphi}{dt} = L\,\frac{di}{dt}$$

Comparison between Capacitor and Inductor

	Capacitor	Inductor
Charging		
Discharging		

Capacitor	**Switch**	**Inductor**
Full	Open	Empty
Empty	Closed	Full

Capacitor

$$v_c(t) = V_{ss} + k\, e^{-\frac{t}{\tau}}$$

- $v_c(t)$: *instantaneous voltage valid for all t*
- V_{ss}*: steady state voltage i.e. when* $v_c(\infty)$
- k*: constant* $= V_C(0) - V_{ss}$
- $\tau = R_{th}C$

$$i_c(t) = C\,\frac{dv}{dt}$$

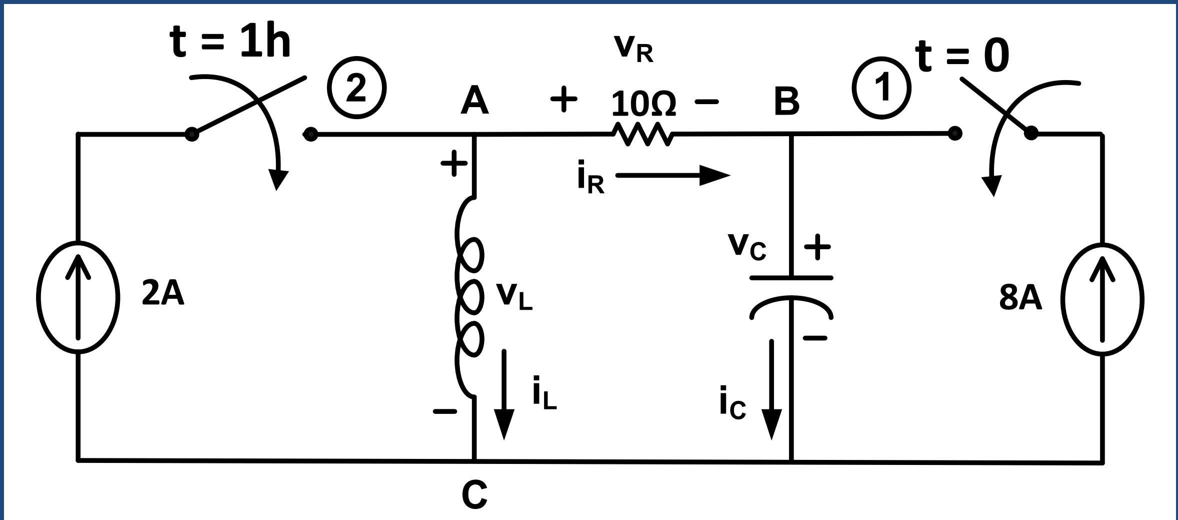
Inductor

$$i_L(t) = I_{ss} + k\, e^{-\frac{t}{\tau}}$$

- $i_L(t)$: *instantaneous current valid for all t*
- I_{ss}*: steady state current i.e. when* $i_L(\infty)$
- k*: constant* $= I_L(0) - I_{ss}$
- $\tau = L/R_{th}$

$$v_L(t) = L\,\frac{di}{dt}$$

Example 1



Switch 1 and 2 were open for a long time, at t = 0 sec, 1 closes then 1h later 2 closes

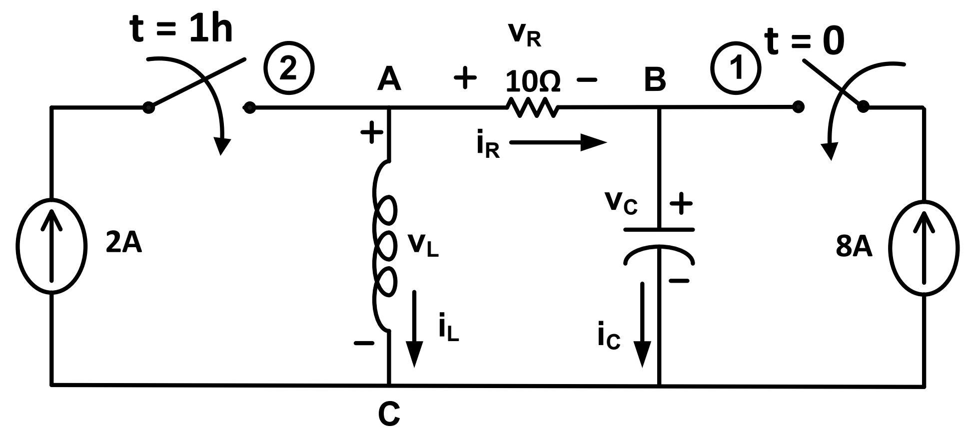
Find: i_R, i_C, i_L, V_R, V_C, V_L

At $t = 0^-$, $t = 0^+$, $t = 1h^-$, $t = 1h^+$, $t \rightarrow \infty$

Solution of ex. 1

t = 0^-

All switches are open => everything is 0



Solution of ex. 1 (cont.)

	i_R [A]	i_C[A]	i_L[A]	v_R [V]	v_C[V]	v_L[V]
t = 0-	0	0	0	0	0	0
t = 0+						
t = 1h-						
t = 1h+						
$t \to \infty$						

Solution of ex. 1 (cont.)

	i_R [A]	i_C[A]	i_L[A]	v_R [V]	v_C[V]	v_L[V]
t = 0-	0	0	0	0	0	0
t = 0+			0		0	
t = 1h-						
t = 1h+						
$t \to \infty$						

Solution of ex. 1 (cont.)

<u>t = 0+</u>

$v_C\,(0+) = v_C\,(0-) = 0$

$i_L\,(0+) = i_L\,(0-) = 0$

Questions:

1. How is switch 1? close
2. How is C? empty d=> SC 
3. How is L? empty => OC 

$v_C = 0V$

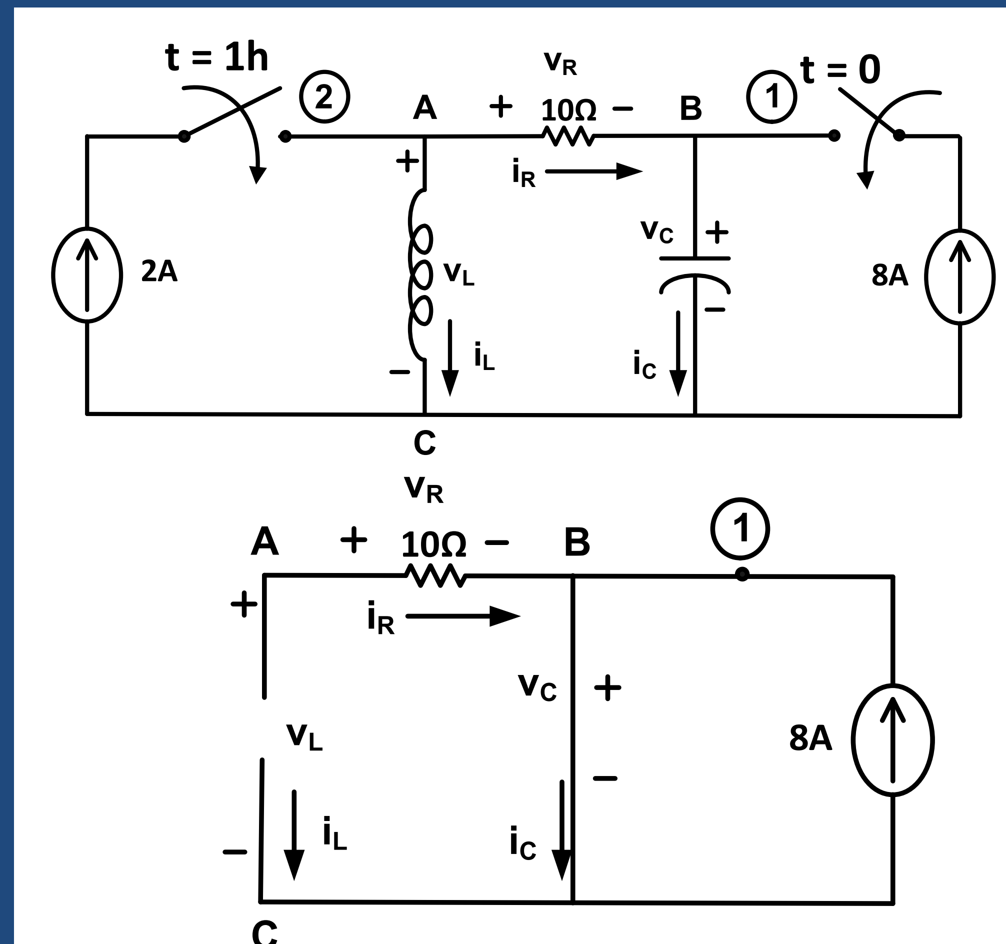
$i_L = 0A$

$i_R = 0A$

$v_R = 0V$

$v_L = v_R = 0V$

$i_C = 8A$



Solution of ex. 1 (cont.)

	i_R [A]	i_C[A]	i_L[A]	v_R [V]	v_C[V]	v_L[V]
t = 0-	0	0	0	0	0	0
t = 0+	0	8	0	0	0	0
t = 1h-						
t = 1h+						
$t \to \infty$						

Solution of ex. 1 (cont.)

<u>t = 1h-</u>

Questions:

1. How are switches 1 & 2? 1 closed, 2
2. How is C? full => OC 
3. How is L? full => SC 

$i_C = 0A$

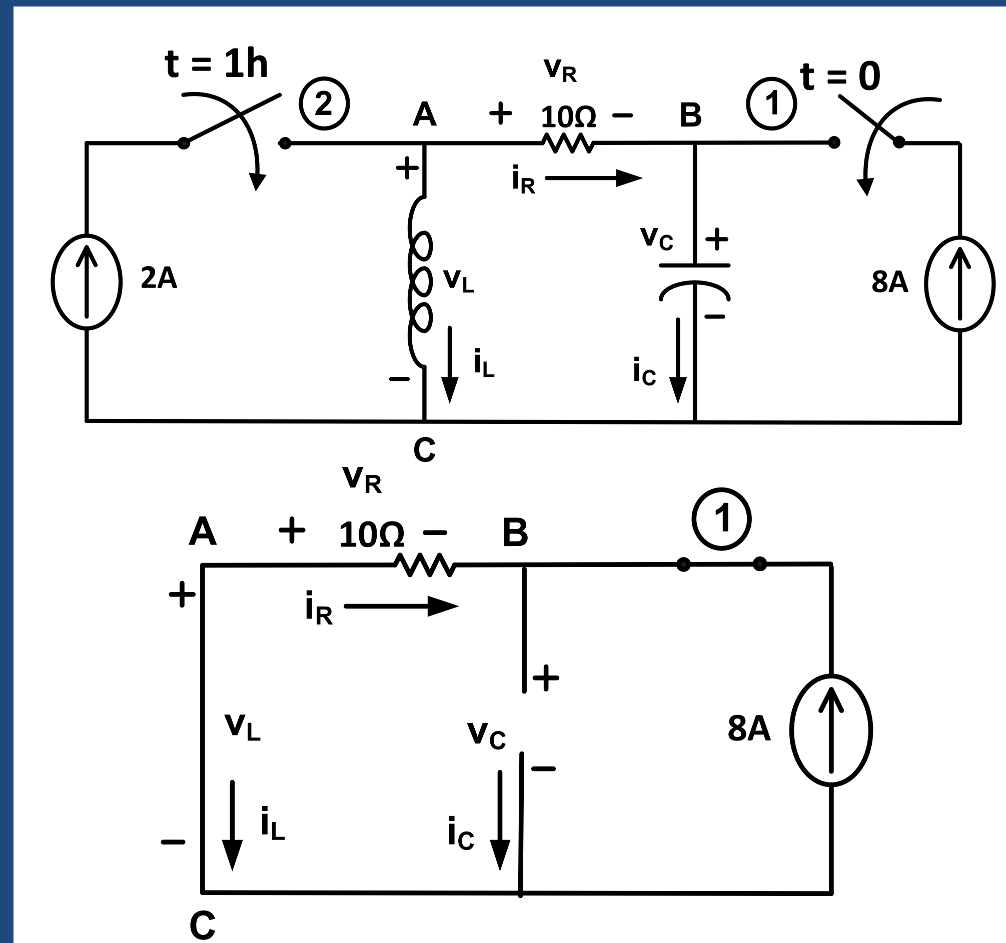
$v_L = V_{AC} = 0V$

$i_L = 8A$

$i_R = -8A$

$v_R = v_{AB} = -80\ V$

$v_C = v_{BC} = 80V$



Solution of ex. 1 (cont.)

	i_R [A]	i_C[A]	i_L[A]	v_R [V]	v_C[V]	v_L[V]
t = 0-	0	0	0	0	0	0
t = 0+	0	8	0	0	0	0
t = 1h-	-8	0	8	-80	80	0
t = 1h+						
$t \rightarrow \infty$						

Solution of ex. 1 (cont.)

	i_R [A]	i_C[A]	i_L[A]	v_R [V]	v_C[V]	v_L[V]
t = 0-	0	0	0	0	0	0
t = 0+	0	8	0	0	0	0
t = 1h-	-8	0	8	-80	80	0
t = 1h+			8		80	
$t \to \infty$						

Solution of ex. 1 (cont.)

t = 1h+

v_C (1h+) = v_C (1h-) = 80V

i_L (1h+) = i_L (1h-) = 8A

Switches 1 & 2 closed

$v_C = v_{BC}$ = 80V

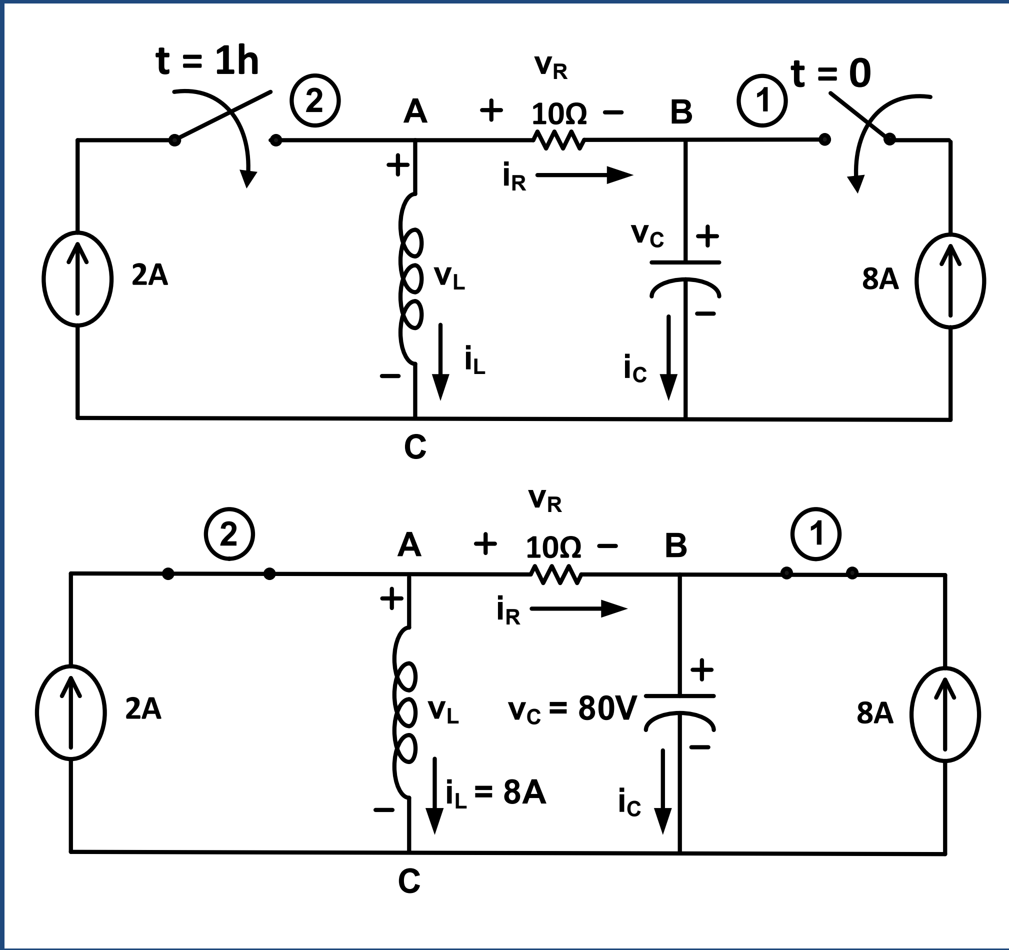
i_L = 8A

$i_R = 2 - i_L = -6$A

$v_R = -60$ V

$i_C = i_R + 8 = 2$A

$v_L = v_{AC} = v_{AB} + v_{BC} = v_R + v_C = 20$V



Solution of ex. 1 (cont.)

	i_R [A]	i_C[A]	i_L[A]	v_R [V]	v_C[V]	v_L[V]
$t = 0-$	0	0	0	0	0	0
$t = 0+$	0	8	0	0	0	0
$t = 1h-$	-8	0	8	-80	80	0
$t = 1h+$	-6	2	8	-60	80	20
$t \to \infty$						

Solution of ex. 1 (cont.)

$t \rightarrow \infty$

Switches 1 & 2 closed

C Full => OC 

L Full => SC 

$i_C = 0A$

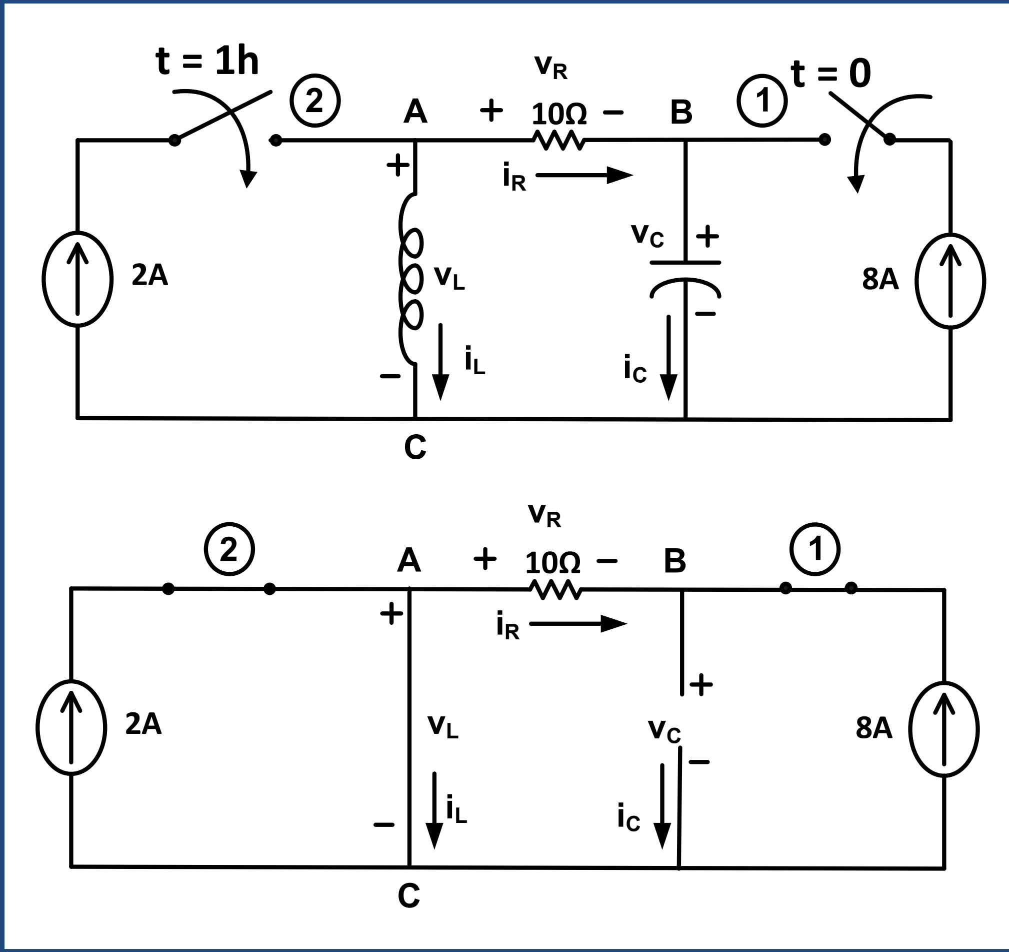
$v_L = 0V$

$i_R = -8A$

$v_R = -80\ V$

$i_L = 8 + 2 = 10A$

$v_C = v_{BA} = -v_R = 80V$



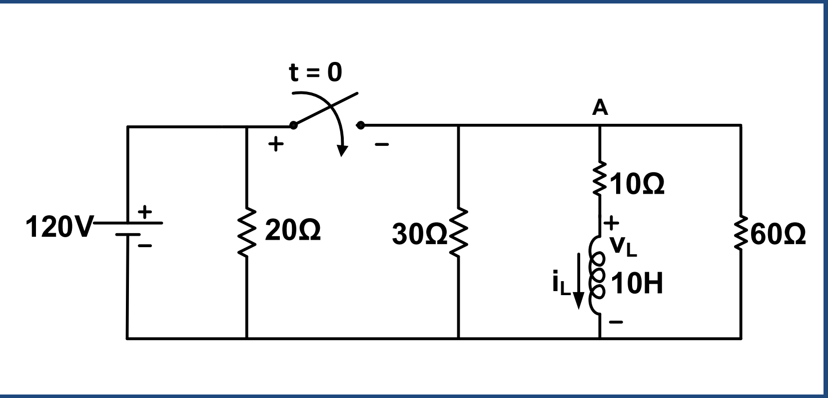
Solution of ex. 1 (cont.)

	i_R [A]	i_C[A]	i_L[A]	v_R [V]	v_C[V]	v_L[V]
t = 0-	0	0	0	0	0	0
t = 0+	0	8	0	0	0	0
t = 1h-	-8	0	8	-80	80	0
t = 1h+	-6	2	8	-60	80	20
$t \rightarrow \infty$	-8	0	0	-80	80	0

Example 2

Switch was open for a long time, at $t = 0$ sec it closes

Find: i_L and v_L for $t \geq 0$



Solution of ex. 2

t = 0-

Switch: open, no source => i_L = 0A

t = 0+

i_L (0+) = i_L (0-) = 0A

t → ∞

1. Switch: closed
2. L: full => SC

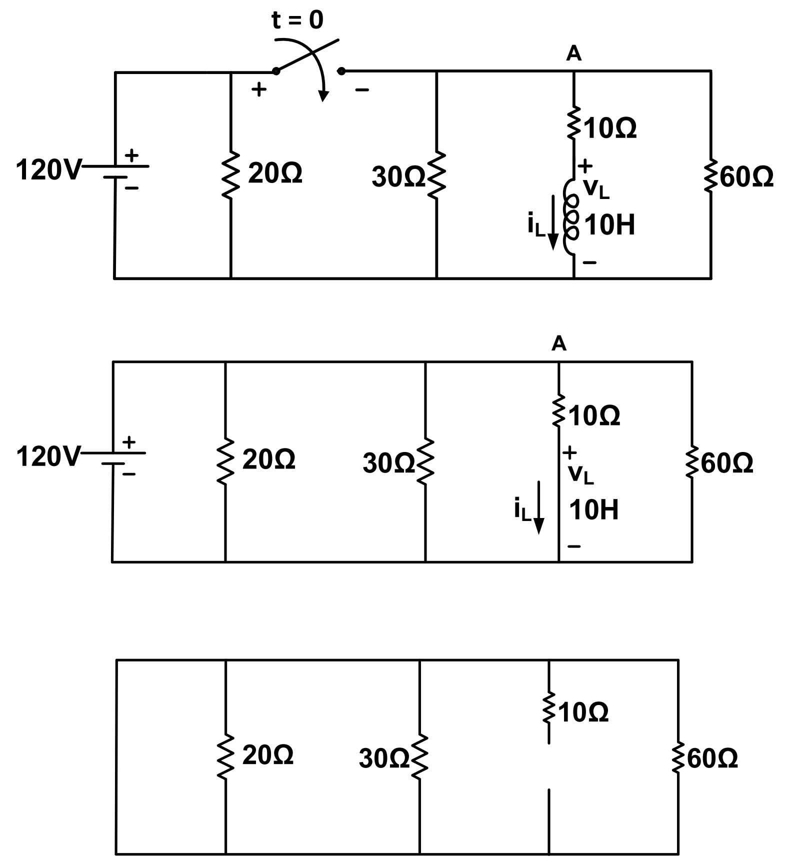
$i_L(\infty) = 120/10 = 12A = i_{SS}$

3. $\tau = L/R_{th}$ = 10/10 = 1 sec
4. $k = i_L(0+) - i_{SS} = 0 - 12 = -12$

=> **For t ≥ 0**:

$i_L(t) = 12 - 12\, e^{-t}$ [A]

$v_L(t) = L\frac{di_L}{dt} = 10\,(-12)\,(-1)\, e^{-t} = 120\, e^{-t}$ [V]



Solution of ex. 2 (cont.)

$i_L\ (0+) = i_L\ (0-) = 0A$

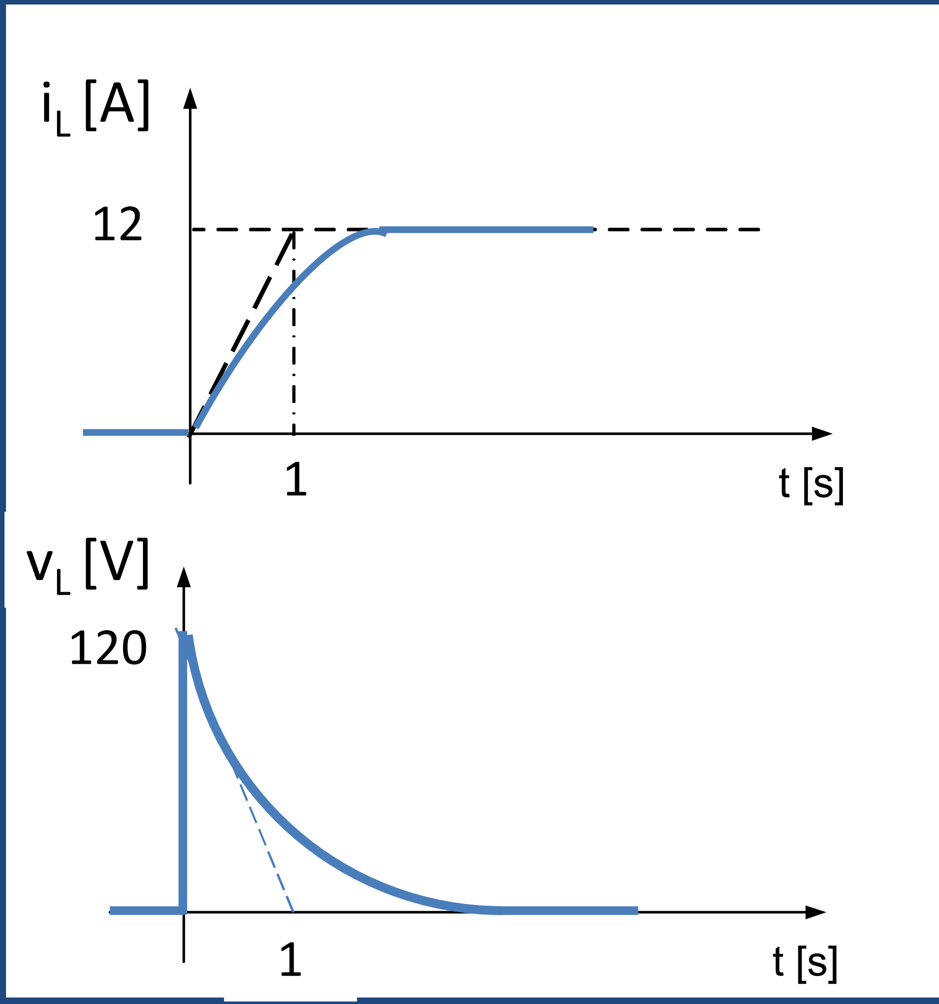
$i_{SS} = 12A$

$\tau = 1$ sec

$v_L\ (0-) = 0V$

$v_L(0) = 120V$

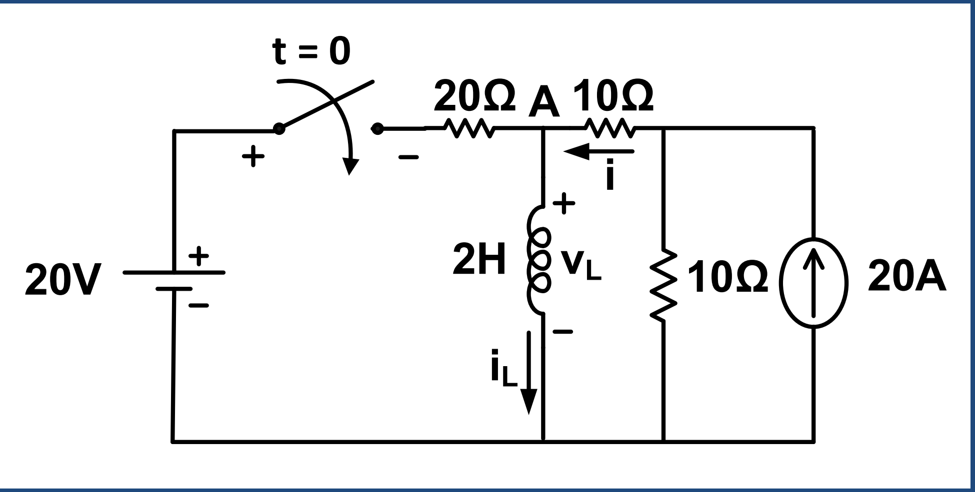
$v_L\ (\infty) = 0V$



Example 3

Switch was open for a long time, at $t = 0$ sec it closes

Find: i_L, v_L and i for $t \geq 0$



Solution of ex. 3

t = 0-

1. Switch: open
2. L: full => SC => i_L = 10A

t = 0+

$i_L(0+) = i_L(0-) = 10A$

t → ∞

1. Switch: closed
2. L: full => SC

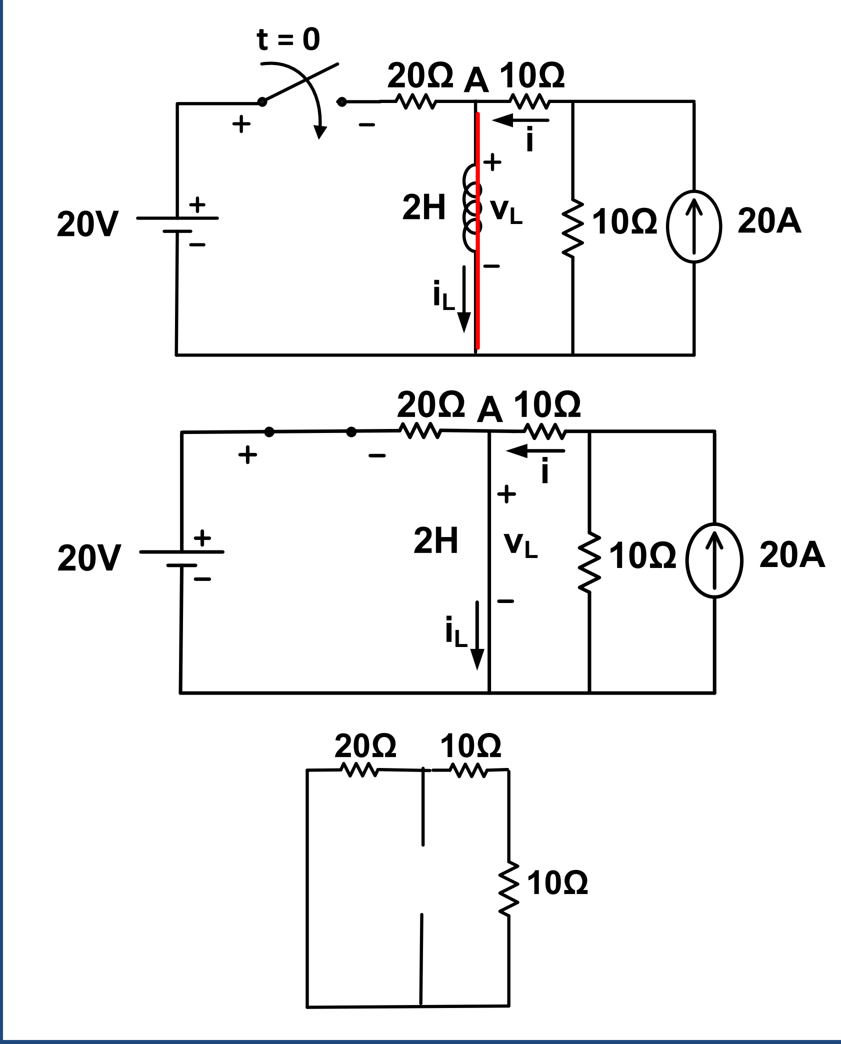
$\Big\}\ i_L(\infty) = i_{SS} = \frac{20}{20} + 10 = 11A$

3. $\tau = L/R_{th} = 2/10 = \frac{1}{5}$ sec
4. $k = i_L(0+) - i_{SS} = 10 - 11 = -1$

=> **For t ≥ 0:**

$i_L(t) = 11 - e^{-5t}$

$v_L(t) = L\frac{di_L}{dt} = 2(-1)(-5)\,e^{-5t} = 10\,e^{-5t}$



Solution of ex. 3 (cont.)

For t ≥ 0: find i

$i_L(t) = 11 - e^{-5t}$

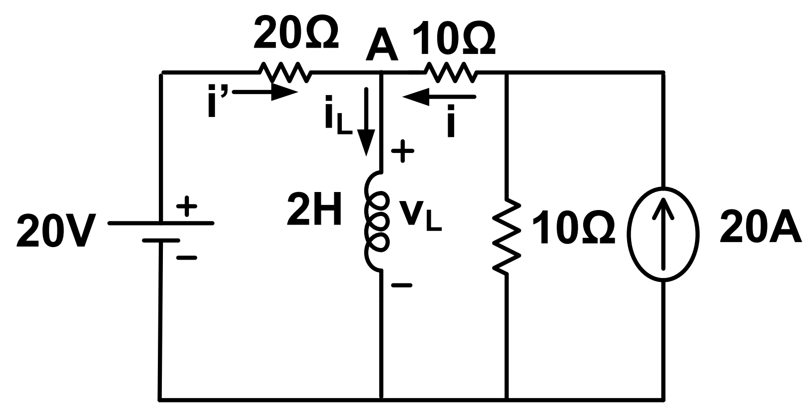
$v_L(t) = L\frac{di_L}{dt} = 10\,e^{-5t}$

KCL at A: $i = i_L(t) - i'$

$i' = \frac{20 - vL}{20}$

$i = 11 - e^{-5t} - \frac{20 - 10\,e^{-5t}}{20}$

$= 10 - \frac{1}{2}\,e^{-5t}$



Solution of ex. 3 (cont.)

i_L (0+) = i_L (0-) = 10A

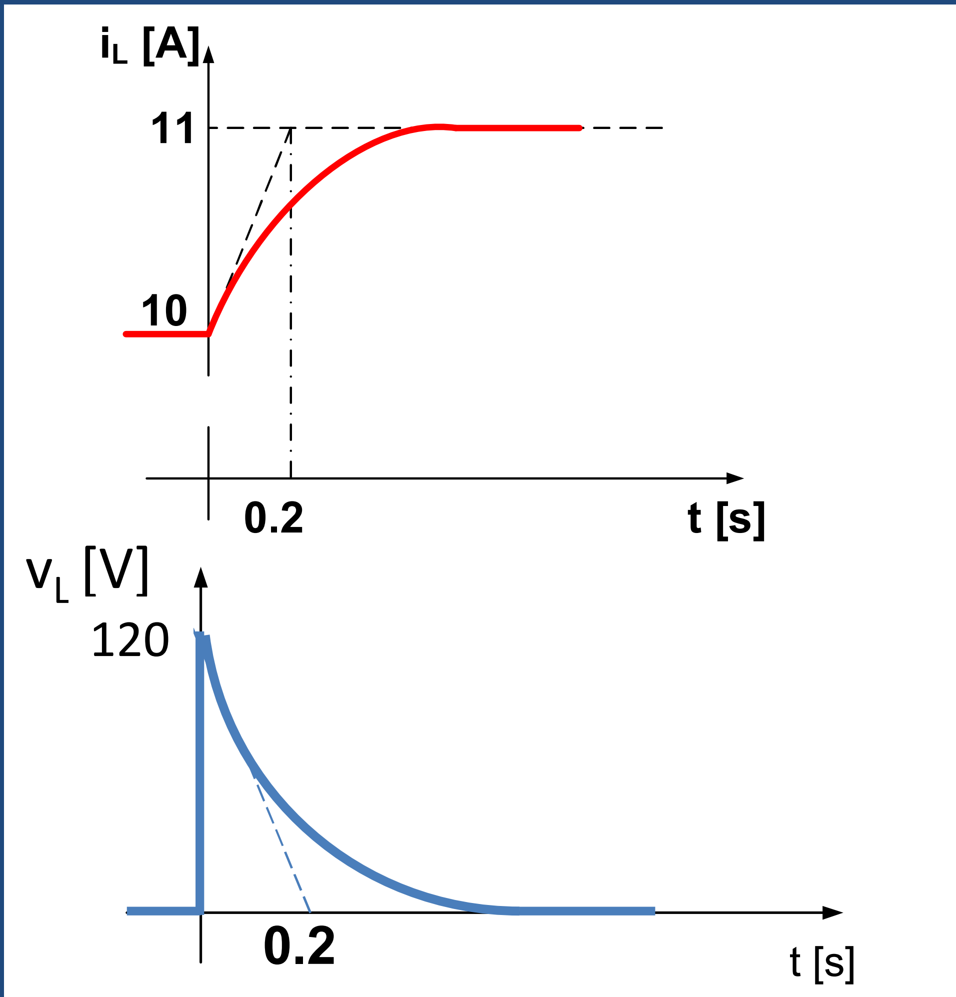
i_{SS} = 11A

τ = 0.2 sec

v_L (0-) = 0V

$v_L(0)$ = 120V

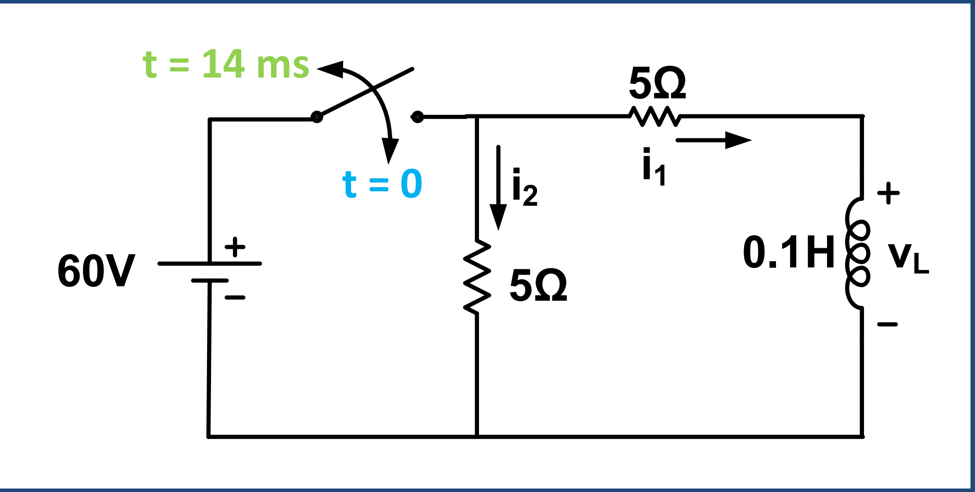
v_L (∞) = 0V



Example 4

Switch was open for a long time, at t = 0 sec it closes then 14ms later it reopens again

Find: i_1, i_2 and v_L for $t \geq 0$



Solution of ex. 4

The way to treat this problem:

<u>*Step 1*</u>*: Ignore the fact that after 14ms switch reopens again i.e., solve the problem as if after t = 0+ the switch is closed 1 for ever.*

<u>*Step 2*</u>*: Calculate* i_L *for t = 14ms*

<u>*Step 3*</u>*: Define a new time scale: t' = t – 14 i.e., when t = 14 ms, t' = 0 sec. Then continue to solve the problem using t' with* i_L(t' =0-) equal to the value calculated in step 2.

<u>*Step 4:*</u> *Summarize all results with respect to the time t*

Solution of ex. 4 (cont.)

<u>t = 0-</u>

1. Switch: open
2. L: empty => $i_L = 0A$

<u>t = 0+</u>

$i_L(0+) = i_L(0-) = 0A$

<u>t → ∞</u>

1. Switch: closed
2. L: full => SC

$\left.\right\}\ i_L(\infty) = i_{SS} = \frac{60}{5} = 12A$

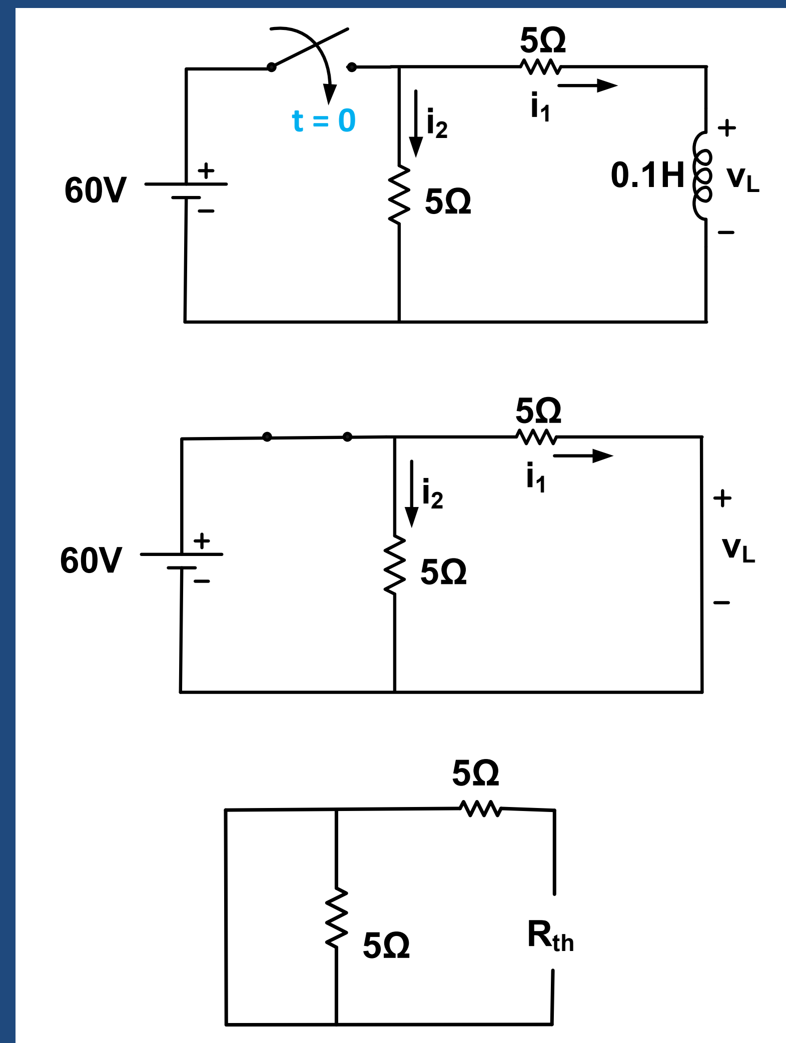
3. $\tau = L/R_{th} = 0.1/5 = \frac{1}{50}$ sec
4. $k = i_L(0+) - i_{SS} = 0 - 12 = -12$

=> **For t ≥ 0**:

$i_L(t) = 12\,(1 - e^{-50t})$

$i_L(14ms) = 12\,(1–50\ e^{-50x0.014}) = 6A$

$v_L(t) = L\,\frac{di_L}{dt} = 0.1\,(-12)\,(-50)\ e^{-50t} = 60\ e^{-50t}$



Solution of ex. 4 (cont.)

Define t' = t – 14ms

t' = 0-

1. Switch: closed => i_L = 6A (see t = 14ms)

t' = 0+

i_L (0+) = i_L (0-) = 6A

t' →∞

1. Switch: open
2. L: empty

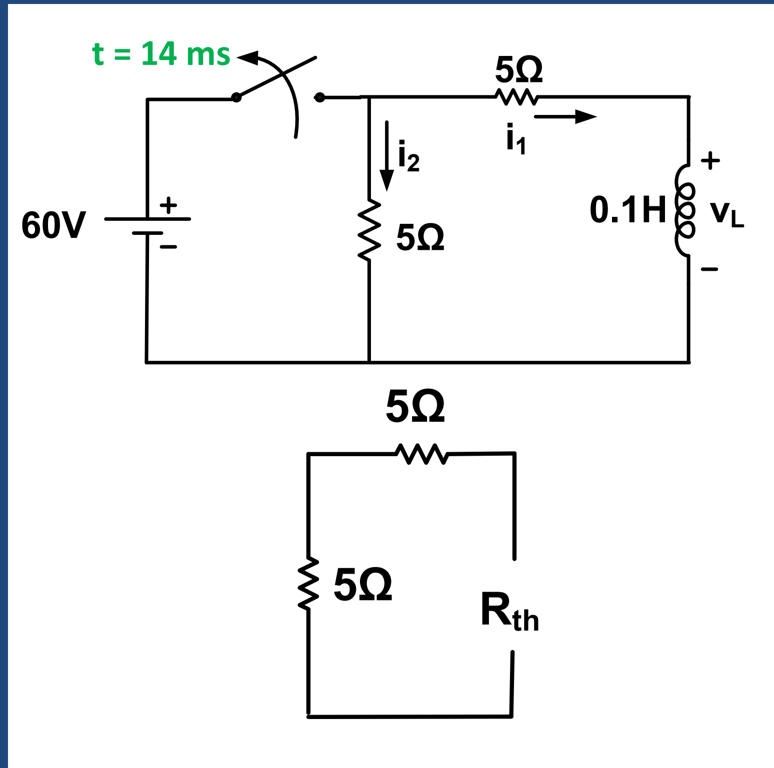
$\Big\}$ $i_L(\infty) = i_{SS} = 0A$

3. τ' = L/R_{th}= 0.1/10 = $\frac{1}{100}$ sec
4. k = $i_L(0+) - i_{SS}$ = 6

=> **For t' ≥ 0**:

$i_L(t') = 6\, e^{-100t'}$

$v_L(t') = L\,\frac{di_L}{dt} = 0.1\ (6)\ (-100)\ e^{-100t'} = -\,60\ e^{-100t'}$



Solution of ex. 4 (cont.)

For $0 \leq t \leq 14$ msec

$$i_L(t) = 12\,(1 - e^{-50t}) \qquad (1)$$

$$v_L(t) = 60\,e^{-50t} \qquad (2)$$

For $t \geq 14$ msec

$$i_L(t) = 6\,e^{-100(t-0.014)} \qquad (3)$$

$$v_L(t) = -60\,e^{-100(t-0.014)} \qquad (4)$$

For $t = 0^-$

$i_L = 0A$
$v_L = 0V$
$i_1 = 0A$
$i_2 = 0A$

For $t = 0^+$

$i_L = 0A$
$(2) \Rightarrow v_L = 30V$
$(4) \Rightarrow v_L = -60V$
$i_1 = 0$
$i_2 = 12A\ (=60/5)$

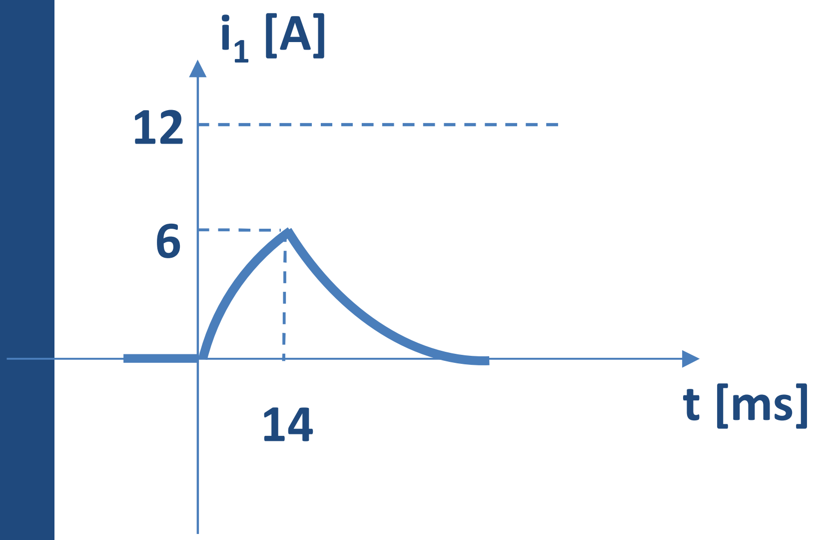
For $t = 14\text{msec}^-$

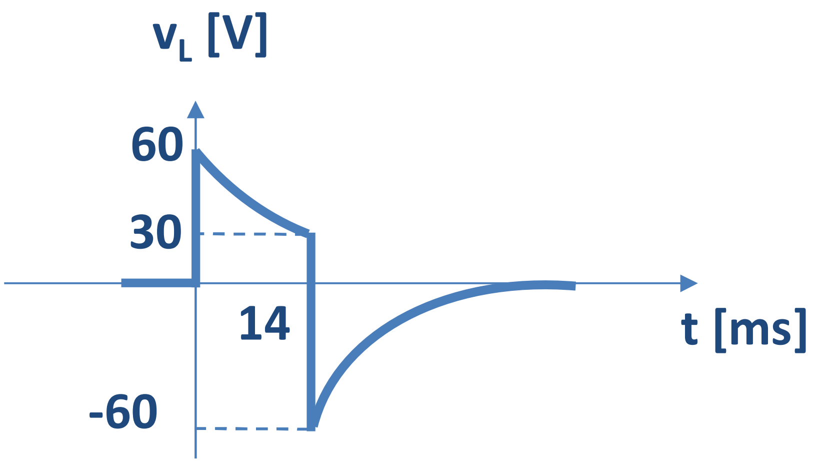
(1) or $(3) \Rightarrow i_L = 6A$
$(2) \Rightarrow v_L = 30V$
$i_1 = 6A$
$i_2 = 12A$

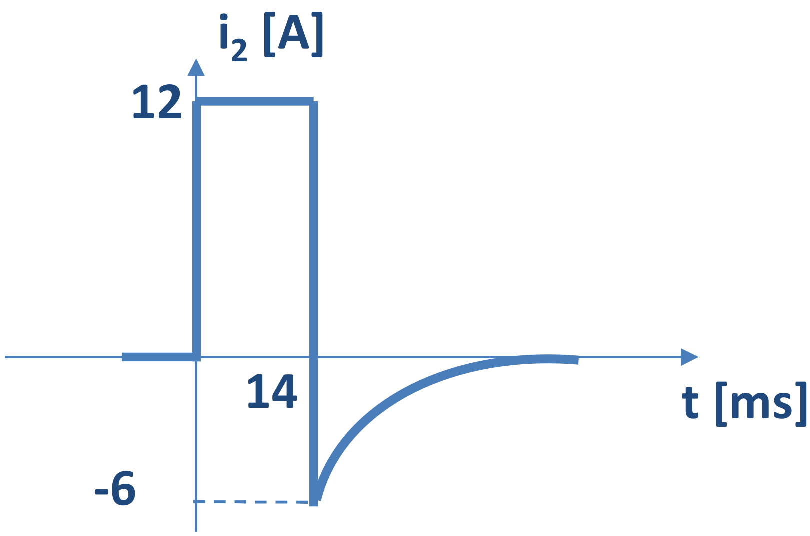
For $t = 14\text{msec}^+$

$i_L = 6A$
$(4) \Rightarrow v_L = -60V$
$i_1 = 6A$
$i_2 = -6A$

Solution of ex. 4 (cont.)







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